A TEXT-BOOK OF SOUND

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PREFACE

The present Text-book of sound is written with the same object as my Text-book of General Physics. It embodies the recently constituted B. A. and B. Sc. Pass Syllabus on Sound of the Calcutta University. It is also expected to cover the B. A. and B. Sc. Syllabuses of other Indian Universities. More advanced portions of the subject are printed in small types which may be omitted in the first reading. To the end of each chapter B. A. and B. Sc. questions of the Calcutta University are appended.

To suit all classes of students two sets of treatments are given ;—one set is algebraic and intended for students who have no knowledge of Calculus, and the other set involves the application of the rudimentary principles of Differential and Integral Calculus.

In preparing the book, almost all the standard works of reference on the subject, such as, Rayleigh's Theory of Sound, Barton's Text-Book of Sound, Wood's Text-Book of Sound, Richardson's Text-Book of Sound, Duncan and Stirling's Text-Book of Physics, Capstick's Text-Book of Sound, Davis' Modern Acoustics, Alexander Wood's Acoustics, etc. have been freely consulted. I take this opportunity to knowledge my indebtedness to the above standard works.

I express my thanks to my friends and colleagues Prof. A. K. Banerjee and Prof. H. P. De, for taking interest in the book and also for correcting some of the proof-sheets. Thanks are also due to Prof. R. K. De of the Scottish Church College and Dr. Sukumar Sircar, D. Sc. of the University College of Science, Calcutta, for helpful suggestions and also to my colleague Prof. A. K. Sen for preparing the index. For going through the proof-sheets, thanks are also due to my friend Mr. Narendra Kumar Das Gupta, M. A., formerly Professor, Anglo-Bengali College, Allahabad.

For the revision of the physiological part of the subject, thanks are also due to our learned Vice principal Dr. B. C. Ghosh, M. A., M. B., B. C., and to my learned friend Dr. K. K. Mukherjee, M. B.

In conclusion I express my gratitude to Prof. Jogesh Chandra Mukherjee, M. A., of the University College of Science, Calcutta, and to Prof. B. N. Sirkar, M. A. of K. N. College, Berhampur, for giving very helpful suggestions towards the improvement of the book.

Physical Laboratory
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PREFACE TO THE THIRD EDITION

Besides sundry changes, a chapter on Longitudinal vibration of rods and bars and supersonics has been added.

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PREFACE TO THE FOURTH EDITION

Fourth edition is almost a reprint of the third edition.

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Some additions have been made in the chapter on Doppler's Principle.

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CONTENTS

A TEXT-BOOK OF SOUND

CHAPTER I

GENERAL IDEAS

1. **Sound : Acoustics**—The word *sound* is generally used in two senses : (1) to denote the *sensation* perceived through the ear,—as we say, we hear a shrill sound, and (2) to denote the *external disturbance* which gives rise to above sensation,—as we say, sound travels faster in solids than in gases.

The branch of physical science which deals with the production, propagation and perception of sound is called *acoustics*.

2. **Production of Sound**—The origin of sound is to be found in the vibratory motion of bodies. The vibratory motion necessary for the production of sound may be of a solid or of a fluid. Thus, when a string of a musical instrument emits a musical sound, the blurred outline of the string shows that it is in a state of vibration. In a trumpet or flute the sound is produced by the vibrations of air.

3. **Limits of Audibility**—Although sound is produced by the vibratory motion of bodies, any vibratory motion is not sufficient to give rise to the sensation of a sound. In order that a sound may be perceived by the ear, the vibrations must be *adequately* quick. The ear can recognise the vibrations of a body provided its frequency of vibration lies

within certain limits called the *limits of audibility*.* The source of sound having this audible frequency of vibration is called a '*sonic*' source. The disturbances produced by it are called '*sonic*' waves. Experiment shows that the lower limit is about 16 vibrations per second although it varies with different persons. Thus, the slow vibratory movements of a pendulum do not give rise to the sensation of sound.

Vibrations slower than the above limit have been recently studied by Esclangon in 1919. Although they are not able to give any sensation of sound still they create disturbances in air which can be detected by manometric flames † and other suitable devices. The disturbances they produce are termed '*infra-sonic*' waves.

The upper limit of audibility lies round 20,000 vibrations per second. It varies for different persons and for any individual diminishes with increasing age. The vibrations of air within a very small whistle are too rapid to produce any sound sensation. The disturbances they create are called '*super-sonic*' or '*ultra sonic*' waves. These waves have got recent application in measuring the depth of ocean due to Langevin (1924 and 1926) in the French system.

4. Propagation of Sound—Experiment shows that sound is ordinarily propagated through air.

If an electric bell is suspended within a bell jar which is greased airtight on the receiver of an air pump, it is found

*Besides the audible range of frequencies, there is an audible range of intensity. The minimum intensity which a sound wave must possess to be audible is called the *threshold intensity of audibility*. Similarly, if the intensity increases a certain value, called the *threshold intensity of feeling*, a sensation of pain is produced in the ear.

† See Art. 114.

that when the jar has been exhausted of air to a sufficient degree the ringing of the bell is almost inaudible. The experiment shows that in order that the sound may be carried from its place of generation to the ear, there must be an uninterrupted communication between the source and the ear. This communication is ordinarily made by the air of the atmosphere.*

Material bodies other than air also prove to be suitable vehicles of sound. The feeble sound of a watch placed at one end of a bench is readily heard by placing the ear at the other end of it although its ticks are inaudible from the same distance within air. The sound of the watch is also clearly heard if it is held between the teeth.

If two persons dive under water and if one of them speaks, his voice is distinctly heard by the other.

These familiar experiments show that sound can be propagated through a solid, liquid or gas but not through an empty space.

We shall return to the mode of propagation of sound in Chapter II.

5. **Preception of Sound**—The production of sound and its propagation are not merely sufficient to give rise to the sensation of sound, unless one possesses a healthy organ of hearing. The disturbances produced by the vibrating source are carried through a proper medium and when they fall upon the drum-skin of the ear, the drum-skin is set to a vibratory motion. The vibrations of the drum-skin are carried through proper vehicles within the ear until they excite the auditory nerves. In this way the sensation of sound is perceived. A detailed study of the ear and the act of hearing will be found in Chapter XVI.

6. **Classification of Sound**—Sound has been divided into two classes : (1) *Noise* and (2) *Musical sound.* The distinction between the two classes seems to be physical as well as physiological. A *musical sound* is characterised by its *periodicity* and *smoothness.* Thus a musical sound possesses regularity of occurrence and it produces an agreeable or pleasant sensation to the ear. Any periodic sound is not necessarily musical. The ticks of a clock, although repeated periodically, do not constitute a musical sound.

A *noise,* on the other hand, is characterised by its lack of periodicity ; it produces, in general, a more or less disagreeable or unpleasant sensation to the ear. The blow of a hammer or any sudden sound of violent character constitutes a noise.

There is however no hard-and-fast line of separation between a noise and a musical sound. Many noises possess a musical character. When a bell is struck repeatedly with a hammer, the noise as well as musical sound is present in the sound emitted. Similarly, some musical sounds are not free from noise. When an organ pipe emits a musical note, the hissing sound of the wind at its mouth is unmusical in character.

The science of acoustics deals with musical sounds, noises being practically excluded for their irregular character.

7. **Classification of Acoustics**—The subject of acoustics includes three branches :

(1) **General Study**—This includes the production of sound, its propagation, wave motion, reflection, refraction, etc.

(2) **Musical Sound**—This branch includes the nature of sounds produced by different musical instruments, the study of intensity, pitch and quality, the musical scale and its temperament, etc.

(3) **Physiological Acoustics**—This part deals with the production of sound by the human beings, the ear, the act of hearing and other studies which are connected with physiology.

CHAPTER II

WAVE MOTION

8. The propagation of sound, light, etc. is inseparably connected with wave motion.

A wave may be defined as a form of disturbance in a medium, due to the particles of the medium executing certain periodic motions about their mean positions. The particles of the medium transmitting waves are not transported permanently from one place to another but they simply execute some periodic motions about their mean positions.

9. **Water Waves**—If some portion of a plane extended water surface is upheaved or some portion is forced down, the action of gravity brings the displaced portions to their undisturbed positions. Light bodies, such as pieces of cork floating on water are not moved onwards by the waves, but they are found to be carried forwards and backwards and partly up and down so as to be left very nearly in the same position.

Lord Kelvin has shown that gravity is not the only controlling force on the displaced water particles. Due to the wave, the surface of water is curved. The surface tension acting upon this curved surface causes it to flatten and thus provides another controlling force on the displaced water particles.

If the waves are long, the surface tension is not an important factor, as the curvature of the displaced portion is small and the waves are chiefly propagated by gravity. These are known as **gravitational waves.** If, on the other

hand, the waves are short, the surface tension is predominant compared to gravity. The waves are chiefly propagated by surface tension rather than by gravity. These waves are called **capillary waves** or **ripples.**

So far as the periodic motions of the individual water particles are concerned, they describe circular paths in vertical planes if the water is *deep*. The radius of the circular path of a water particle becomes shorter and shorter, the greater the depth of the particle below the surface. On the other hand, if the water is *shallow*, the water particles execute elliptic motion in vertical planes, the vertical axis of the ellipse decreasing with increase of depth of a particle.

10. Transverse Waves—Take a solid india-rubber cord (fig. 1) about 30 feet long and suspend it vertically from a hook in a wall. If the lower end of it is moved side-ways to and fro, a disturbance in the form of a wave will travel along the cord. Here the motion of separate parts of the cord is *transverse* or at right angles to [redacted] wave propagation. The waves thus set up are called *transverse waves*.

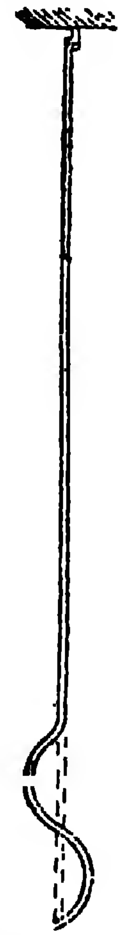

FIG. 1

11. Longitudinal Waves—Make a long spiral of copper wire by winding it round a long wide glass tube. Suspend the spiral from a wooden frame as shown in fig. 2 by threads. The bifilar suspension prevents any lateral swing of the spiral. If a smart push is given to one end of the spiral, a pulse of compression will travel along the spiral. Similarly, on giving a smart pull a pulse of rare faction

will travel along it. By giving the end a push followed by a pull a wave of compression and rarefraction will travel along the spiral. The motion of individual turns of the

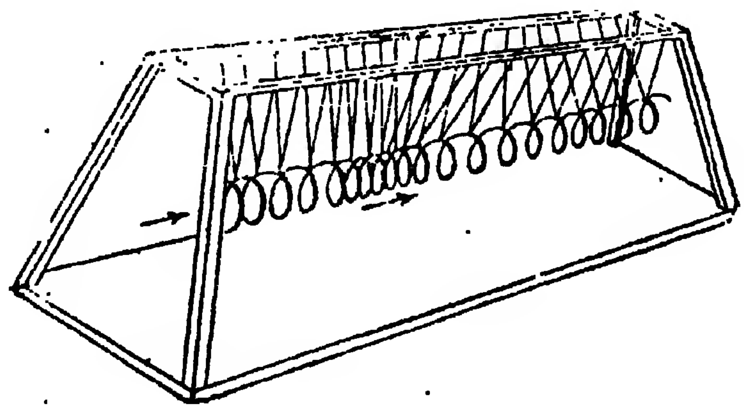

FIG. 2

spiral is thus along the direction of wave propagation. These waves are called *longitudinal waves*.

12. **Propagation of Light and Sound**—The rectilinear propagation of light is inferred from the formation of geometrical shadow cast by an obstacle held before a source of light. It was subsequently observed that if the obstacle is a narrow one, such as the end of a needle, a certain amount of light is obtained within the geometrical shadow of the obstacle. Thus light bends round the corner of a narrow obstacle. The same phenomenon is exhibited by sound which shows appreciable bending round any obstacle. This phenomenon of bending round corners shown by light and sound is known as **Diffraction.***

Another class of phenomenon which is observed in connection with the propagation of light is the production of illumination and darkness at alternate places by two sources of light sending out light radiations under certain

* **For a detailed account see Art. 98.**

conditions. A similar phenomenon is shown by two sources of sound, sending sound radiations under the same conditions, which produce alternate places of sound and silence. This phenomenon is known as **Interference.***

Both the phenomena of diffraction and interference shown by light and sound can only be properly explained if they are supposed to a travel in the form of waves.

In order that the propagation can take place in form of waves, there must be some medium in which the waves will be formed. The silence of a bell ringing in vacuo, shows that the medium for sound propagation is ordinarily air ; although sound can also be propagated through a solid or a liquid medium. But an ordinary electric glow lamp, whose incandescent filament sends out light through the bulb which is practically devoid of air, shows that air is not the vehicle of light waves. Since a medium is essential for the propagation of light waves, the medium, supposed to exist, is ether which pervades throughout the universe. Although direct evidence on the existence of ether is lacking, still no space can be freed from it.

The phenomena of interference and diffraction show that light and sound are propagated in form of waves. Let us now enquire into the nature of these waves.

Let us suppose that sound waves propagating through air are transverse. Let the waves travel along AB (fig. 3) and let L_1, L_2, L_3, etc. be the different layers of the medium. If L_1 moves up in course of its vibratory motion perpendicular to the direction of wave propagation, it must exert a force on the adjacent layer L_2 dragging the

* For a detailed account see Chapter XIII.

latter upwards so that a wave may be formed in the medium. Such a force can arise out of *shear elasticity* of

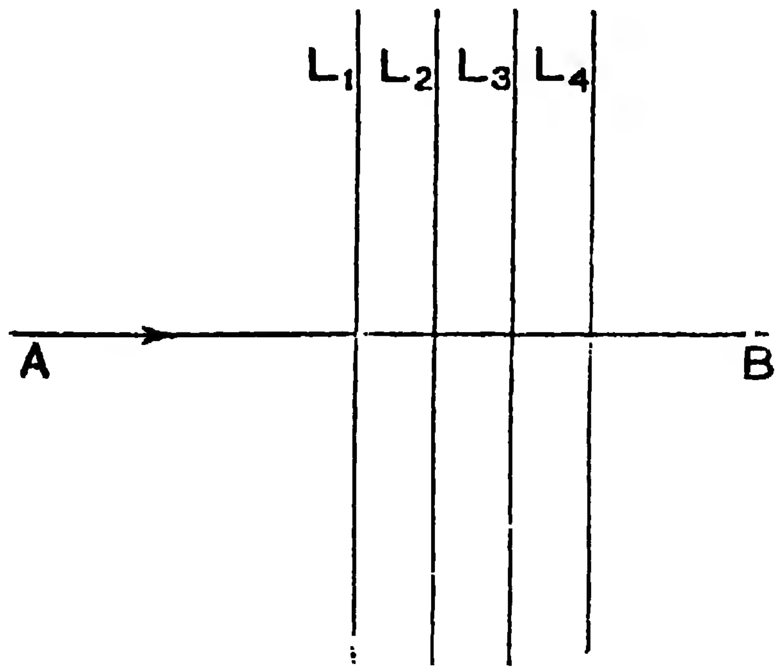

FIG. 3

the medium. But air or any other fluid medium is not possessed of shear elasticity. The only force which the layer L_1 can exert in the present case is the viscous force due to the *relative motion* between L_1 and other layers of the medium. Such a viscous force which always *opposes* the relative motion tends to drag the adjacent layer L_2 up and ceases to exist when the layers are brought relatively to rest. Similarly, the motion of L_2 reacts upon the adjacent layer L_3 and drags it up until they are brought relatively to rest. Again, when the layer L_1 moves down, the viscous force of L_2 *opposes* its motion ; similar is the case of other successive layers. This process of transmission of the wave is thus associated with a great loss of kinetic energy. Thus although a transverse wave may be created in a fluid medium it cannot be propagated very

far. Since with sound waves there is no evidence of such unusual subsidence, they cannot be transverse.

If, on the other hand, the waves are longitudinal, the layers move to and fro along the direction of wave propagation. Thus when L_1 moves in the direction AB, it makes compression in the forward direction and rarefaction behind it. Its motion is controlled by the layers of compression before it and by the layers of rarefaction behind it. The controlling force arises out of the *volume elasticity* of the medium. The kinetic energy of the moving layer L_1 is stored up as the potential energy of compression. During the backward motion of the layer L_1 in the direction BA its motion is *helped* by the layers of compression and rarefaction created by it before, until it regains its normal undisplaced position. The potential energy of compression partly reappears as kinetic energy of the layer L_1 during return journey and enables it to be carried beyond its normal position producing a region of compression before it and a region of rarefaction behind it. The potential energy is also partly shared by the adjacent layer L_2 and causes L_2 to move forward. Thus air or any other fluid which is possessed of volume elasticity, proves a suitable medium for the propagation of longitudinal waves.

It may be observed that a medium which is possessed of shear elasticity like a solid, can transmit the transverse waves. The kinetic energy of a moving layer is stored up as potential shear energy which reappears as the kinetic energy during the return journey of the layer.

The sound waves which are ordinarily propagated through air are thus longitudinal. The light waves, on the other hand, owing to their transverse character, require for

their propagation a medium possessed of shear elasticity.* This medium is called ether.

Another conclusive evidence on the character of light and sound waves is shown by the phenomenon of polarisation.* This phenomenon can be shown by transverse waves alone and not by longitudinal waves. Light waves which show the phenomenon are thus transverse whereas sound waves which fail to show the phenomenon are necessarily longitudinal.

13. Harmonic Waves—If the particles of a medium execute simple harmonic motions † during the propagation of waves through it, the waves are called *harmonic waves.* When a tuning fork vibrates in air, the particles of air are thrown into simple harmonic oscillations; the longitudinal waves thus generated are referred to as of the *simple harmonic type* and are known as harmonic waves.

Examples

1. Describe a method of compounding two vibrations at right angles to each other. Work out the case in which the phase difference is half.

2. What is the resultant effect of two vibrations in the same direction? Investigate all the different cases that arise. (*C. U. 1924*)

* For a study of the phenomenon, the student is referred to any standard book on Physical Optics.

† For a detailed account of Simple Harmonic Motion the reader is referred to 'A Text-Book of General Physics' by the author or to 'A Text-Book of Sound', Barton.

CHAPTER III

GRAPHICAL REPRESENTATION OF TRANSVERSE AND LONGITUDINAL WAVES

14. Graphical Representation of a Transverse Wave—A transverse wave of the simple harmonic type can be easily represented by a graph.

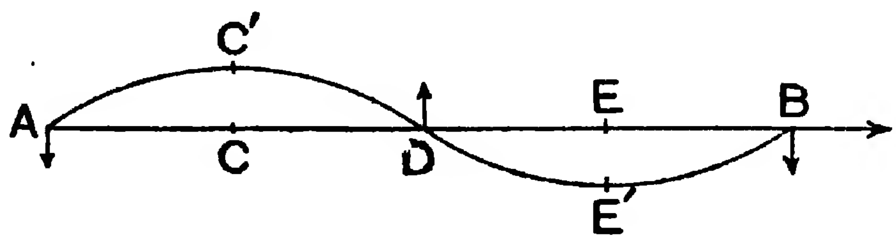

Fig. 4.

Let the transverse waves travel in the direction AB (fig. 4) in a medium. The particles of the medium execute S. H. M.'s perpendicular to AB. Let a particle of the medium situated at A just complete one vibration by starting from its undisplaced position A moving downwards to the maximum displacement, then going through A to the maximum displacement in the upward direction and finally reaching its undisplaced position A, just tending to repeat its journey downwards. Suppose during this time which the particle at A has taken to execute one complete oscillation, a length AB of the medium is disturbed. We shall study the displacements and states of motion of various particles of the medium lying in the region AB at this instant.

If we consider a particle of the medium at C, such that the distance of C from A is $\frac{1}{4}$ AB, the disturbance

reached C at a time $\frac{T}{4}$ later than it reached A, (where T is the period of vibration of the particle). Therefore the phase of vibration of C will be $\frac{T}{4}$ behind that of A. Hence the particle at C must have finished its downward journey starting from its undisplaced position and is displaced upwards to C' to the maximum extent.

Similarly if we consider a particle of the medium at D, such that $AD = \frac{1}{2}AB$, the disturbance reached D at a time $\frac{T}{2}$ later than it reached A. Hence the phase of vibration of the particle at D will be $\frac{T}{2}$ behind that of A. The particle at D has thus completed its downward motion and is just tending to pass through its undisplaced position D in the upward direction.

A particle of the medium at E, such that $AE = \frac{3}{4}AB$, has been disturbed at a time $\frac{3}{4}T$ later than the particle at A and its phase of vibration is $\frac{3}{4}T$ behind that of A. In other words, it is displaced to the maximum extent in the downward direction to E'.

A particle at B is disturbed at a time T later than the disturbance reached A and hence its phase of vibration is T behind that of A. In other words it is going to move downwards through its undisplaced position B.

The *actual* positions of the particles of the medium are shown by the wavy curve $AC'DE'B$, which resembles a *sine* or a *cosine* curve and is called a *wave curve.*

The wave curve, besides representing the *actually* displaced positions of the particles of the medium, reveals some other properties of the displaced particles.

For example,—(1) at points A, D, B, etc. the particles are passing through their mean positions and thus possess the maximum velocity. While at points C', E', etc. the particles are displaced to the maximum extents and the velocities of these particles are zero. If tangents are drawn to the curve at A, D and B, these will include the maximum angle with AB. Similarly, the tangent at C' or E' will include no angle with AB. Hence the *slope* of the curve at any point represents the velocity of the moving particle at that point. (2) The *rate of change of slope* at C' or E' is maximum as the curve bends at these points. The particles at these points possess the maximum acceleration. Similarly, the rate of change of slope at A, D or B is zero as the curve is straight at these points. The acceleration of the particles of these points is zero. Thus the *rate of change* of slope at any point on the curve represents the acceleration of the moving particle at the point.

15. Wave-length—The distance AB (fig. 4) through which the wave travels during one complete vibration of one of the vibrating particles of the medium is called the *wave-length* (represented by λ) of the wave.

It will [redacted] that the particles at A and B are in the same phase of vibration as they are moving through their undisplaced positions in the *same direction.* Any other particle, which will be in this phase of vibration, will be at a distance AB or its multiple from A or B. *Thus the wave-length is the shortest distance between any two particles in the same phase of vibration.*

The particles at A and D are passing through their undisplaced positions in the opposite directions, hence they

are in opposite phases of vibration.* Further, the wave travels through the distance AD in one-half of the time of vibration of a particle of the medium. Hence the least distance between two particles in opposite phases of vibration amounts to half the wave-length. Similarly, the distance CE amounts to half the wave-length and the distance AC or CD is equal to one-quarter of the wave-length †

16. Crest and Trough—A point on the wave such as C' (fig. 4) at which the particle is displaced to the maximum extent in the positive direction is called a *crest*, while a point on the wave, such as E', at which the particle is displaced to the maximum extent in the negative direction is called a *trough*. The terms *crest* and *trough* are more often used in connection with water waves and occasionally with transverse waves.

17. Amplitude—The maximum displacement of a particle on the wave from its undisplaced position is called the *amplitude* of the wave. Thus in fig. 4, the maximum displacement which a particle on the wave can have amounts to CC' or EE'. This distance CC' or EE' is called the amplitude of the wave.

18. Period—If T represents the period of vibration of a particle on the wave, the wave is propagated through a distance equal to the wave-length in time T. The time T is

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* See A Text-Book of General Physics by the author art. 88.

† The wave-length (λ) or its fraction has often been used to denote the phase of a vibrating particle when waves are proceeding from it, in addition to the usual ways of measuring phase (see the Author's Text-Book of General Physics, Art. 89). Thus the vibrating particles at A and D in fig. 4, which are separated by a distance $\frac{\lambda}{2}$, have opposite phases of vibration or phase-difference of π. Similarly, the vibrating particles at A and B, which are separated by a distance λ, have the same phase of vibration or a phase-difference of 2π.

called the *period* of the wave and is defined as *the time which the wave takes to travel a distance equal to its wave-length.*

19. Velocity of Propagation—The distance through which a wave travels in one *second* in a medium, is called the *velocity of propagation* of the wave in that medium. It depends on the properties (elasticity and density) of the medium and is represented by V.

20. Frequency—During each complete vibration of a particle of the medium or of the source sending out the waves, one wave is formed. If n represents the frequency of vibration, *i. e.* the number of vibrations executed in one second by a particle of the medium or by the source, n waves are formed in one second. The number of waves n formed in one second, is called the frequency of the wave. The wave generated by a source at the beginning of a second will travel through a distance V in one second and the wave generated at the end of the second will lie just near the source. The n waves formed by the source in one second will thus be contained within a length V, where V is the velocity of propagation. Thus the frequency of a wave may also be defined as *the number of waves contained within a length equal to the velocity of propagation.*

21. Relation between Period or Frequency, Wave-length and Velocity of Propagation—If T represents the period of the wave, the wave travels through a distance equal to the wave-length λ in the time T. Again, if V be the velocity of propagation of the wave, the wave travels through a distance $V.T$ in the time T. Hence, we have the important relation.

$$VT=\lambda \quad \ldots \quad \ldots \quad (1)$$

The relation can also be expressed in another way. If n be the frequency of the wave, n is the number of vibrations executed by a particle on the wave in one second. Again, T is the time required for a single oscillation. Hence,

$$nT=1 \quad \text{or} \quad T=\frac{1}{n}$$

Substituting for T in (1), we have

$$n\lambda = V. \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad (2)$$

22. Graphical Representation of a Longitudinal Wave : Displacement Curve—A longitudinal wave of the simple harmonic type cannot be represented graphically in the same way as a transverse wave. For, in a longitudinal wave, the displacements of the various particles take place along the direction of wave propagation and a similar graphical representation, as in the case of a transverse wave showing the actual positions of the displaced particles will give a straight line along the direction of propagation of the wave.

But there is a very valuable graphic method of representing the longitudinal waves of the simple harmonic type, which reveals the state of affairs in different parts a wave.

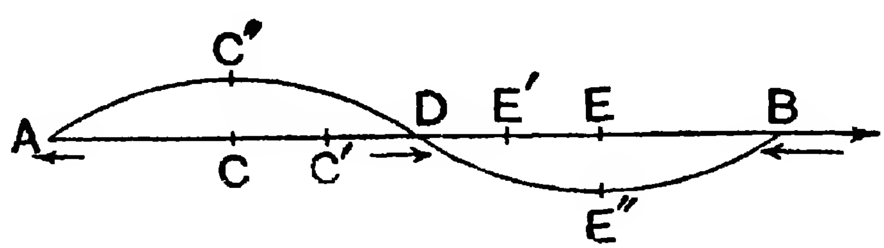


FIG. 5

Let longitudinal waves of the simple harmonic type be propagated along the straight line AB (fig. 5) in a medium.

A particle on the wave executes S. H. M. along AB. Let the disturbance travel the distance AB during one complete oscillation of a particle on the wave.

Suppose, a particle at A has just completed one oscillation, by starting from its undisplaced position A, moving leftward to the maximum displacement, then going through A to the maximum displacement rightward and finally reaching A, just going to repeat its journey leftward. We shall study the displacements and states of motion of various particles of the medium lying in the region AB at this instant.

If we consider a particle at C, the distance of which from A amounts to $\frac{1}{4}AB$, the disturbance reached the particle at a time $\frac{T}{4}$ later than it reached A : where T is the period of vibration of the particle. The phase of vibration of the particle at C is thus $\frac{T}{4}$ behind that of A. Hence, the particle at C must have finished its leftward journey and is displaced rightward to the maximum extent to C'. To determine the displacements of the particles between A and C, we see, that the particle at A is undisplaced while at C is displaced in the rightward direction to the maximum, the displacements of the particles within this region will evidently be in the rightward direction, increasing gradually as the particle is nearer to C. To determine their directions of motion, we see that the particle at A is moving leftward with the maximum velocity while the particle at C is at rest, the velocities of the intermediate particles will be leftward, decreasing gradually as we pass from A to C. The acceleration of the particles can also be studied from their displacements. We know that in the

simple harmonic motion, the acceleration of the particle is proportional to its displacement and is directed oppositely to the displacement. Hence, the particle at A has no acceleration as it passes through its mean position, the particle at C has the maximum acceleration leftward as it is displaced to the maximum amount in the rightward direction. The acceleration of the particles between A and C are in the leftward direction increasing in magnitude from A to C.

At a point D, the disturbance reached at a time $\frac{T}{2}$ later than it reached A as D is situated at a distance $\frac{1}{2}AB$ from A. Thus the particle at D has completed its leftward journey and is on the point of going to move rightward. Hence, the displacement of the particle at D is zero but it possesses he maximum velocity in the rightward direction. The articles within C and D are displaced in the rightward direction, but the displacement decreases from C to D. The velocity of the particle at C is zero and that at D is maximum in the rightward direction. Hence, the particles in the intermediate region CD are moving rightward, the velocity gradually increasing as we pass from C to D. consideration of the displacement at once shows that the acceleration of the particle at C is maximum leftward while that of the particle at D is zero. The accelerations of the intermediate particles are in the leftward direction decreasing in magnitude from C to D.

At a point E situated at a distance $\frac{3}{4}AB$ from A, the disturbance reached at a time $\frac{3}{4}T$ later than it reached A. The phase of vibration of the particle at E is thus $\frac{3}{4}T$ behind that of A. Hence, the particle at E is displaced to the

maximum extent in the leftward direction to E'. Thus its velocity is zero. The displacement of the particles in region DE increase in magnitude from zero to the maximum value in the leftward direction as we pass from D to E. The velocity of the particle at D is maximum rightward, that of E is zero and of any other particle in the intervening portion the velocity is along the rightward direction and becomes less, the nearer the particle is to E. A consideration of the displacements of the various particles within the region DE at once shows that the acceleration of any particle is in the rightward direction increasing in magnitude from zero at D to the maximum value at E.

At the point B, the disturbance reached at a time T later than it reached A. The phase of vibration of the particle at B is thus identical with that of the particle at A. The particle at B is thus moving leftward through its mean position. The displacements of the particles in the region EB are leftward decreasing in magnitude from a maximum value at E to a zero value at B. The velocities of the particles in this region increase in magnitude from a zero value at E to a maximum value at B, directed in the left-irection. A consideration of the displacements of various particles in the region EB shows that the accelerations of the various particles in this region change from a maximum value at E to a zero value at B. Further, since the displacements in this region are in the leftward direction, the accelerations are directed rightward.

Let from the undisplaced position of each particle on the line AB, a normal to AB be drawn proportional in length to the displacement of the particle, a displacement

in the rightward direction being represented by a normal drawn upwards, while a displacement in the leftward direction being represented by a normal drawn below the line AB. If the ends of these normals are joined by an even curve, it will resemble a *sine* or a *cosine* curve. This curve is known as the *displacement curve* of the longitudinal wave. It will be understood that the curve *does not* represent the *actual* positions of the vibrating particles as in a transverse wave, but is a conventional diagram in which the displacements of various particles are plotted against their undisplaced positions. Such a curve is shown by the wavy line $AC''DE''B$ in fig. 5.

23. Properties of the Displacement Curve—The displacement diagram reveals all the properties, *viz.* velocity, acceleration, states of compression or rarefaction, etc., of the particles in the medium.

For example—(1) the *actual* positions of the particles on the line AB can be determined from the displacement curve. To determine the actual position of a particle on AB, draw a normal to AB from the undisplaced position of the particle to meet the displacement curve. If the normal is *above* the line AB, the displacement is towards the and if the normal is *below* the line AB, the displacement is towards the *left*. Further, the length of the normal is proportional to the *magnitude* of the displacement. (2) The *slope* at any point on the displacement curve represents the *velocity* and (3) the *rate of change of slope* represents the *acceleration* of the corresponding particle on AB.

The slope of the displacement curve decreases as we pass from A to C'' (fig. 5), the velocity of a particle, which is proportional to the slope, decreases from A to C.

The rate of change of slope increases as we pass from A to C''. The acceleration of a particle, which is proportional to the rate of change of slope, increases from A to C.

The slope of the curve is zero and the rate of change of slope is maximum at C'' which show that the velocity of the particle at C which is displaced to C' is zero and its acceleration is a maximum.

Again, the slope of the curve gradually increases but the rate of change of slope decreases from C'' to D, showing that the velocity of a particle increases but its acceleration decreases as we pass from C to D. At D the slope of the curve is a maximum but the rate of change of slope is zero showing that the velocity of the particle at D is a maximum but its acceleration is zero. After D the slope of the curve decreases and the rate of change of slope increases up to the point E'', which reveals the fact that the velocity gradually decreases but the acceleration increases from D to E. At E'' the slope is zero but the rate of change of slope is a maximum showing that the velocity of the particle at E which is displaced to the maximum extent is zero but its acceleration is a maximum. In the part E'' B of the curve ope again increases until it becomes a maximum at B but the rate of change of slope decreases until it is zero at B. These facts show that the velocity of the particle increases from E to B until it reaches a maximum at B, on the other hand the acceleration decreases from E to B until it has a zero value at B.

(4) The displacement diagram also shows the states of compression and rarefaction of the medium along AB. The particles to the right of A and B are displaced in the rightward direction and those to the left of A and B are

displaced in the leftward direction. Thus A and B are places of maximum rarefaction. The particles to the right of D are displaced leftward while those to the left of D are displaced rightward, showing that D is a place of maximum compression. In the immediate neighbourhood of C the particles on both sides of it are equally displaced in the rightward direction. Similarly, in the immediate neighbourhood of E the particles on both sides of it are equally displaced in the leftward direction, showing that C and E are places of normal pressure.

24. Figure 6* shows the actual positions, velocities and accelerations of the particles and also the variations of pressure at different points on a longitudinal wave in

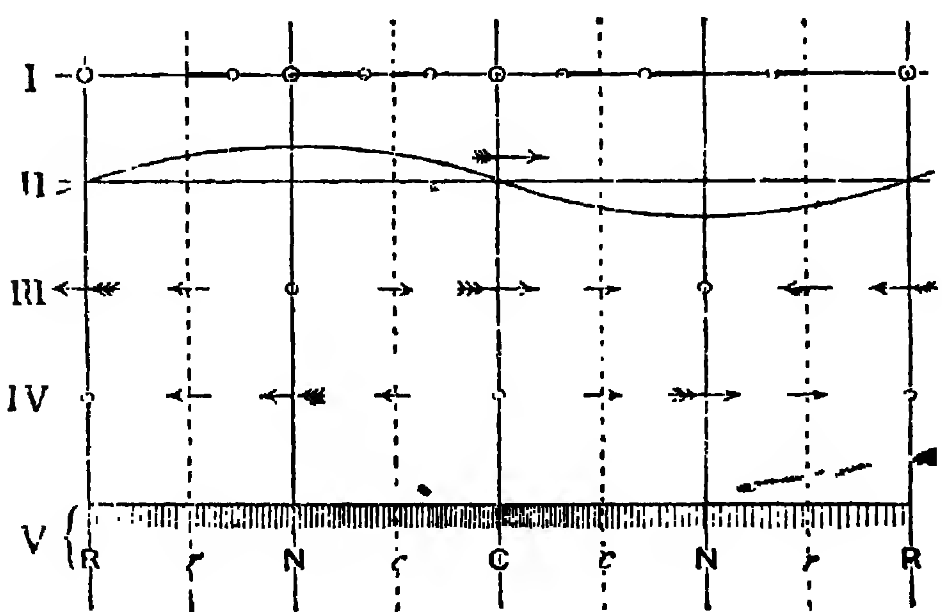


Fig. 6

a convenient manner. The wave is supposed to travel rightward. The first horizontal line (line 1) shows the

* The figure is taken from Barton's Text-Book of Sound with slight modifications.

actual positions of the particles in the medium in course of their vibratory motion. The second line (line II) represents the conventional displacement curve for the longitudinal wave. The arrows on the third line (line III) give the direction and a rough relative measure of the magnitude of velocity of the various particles in vibration, while the arrows on the fourth line (line IV) give the direction and a relative measure of the acceleration of the particles. In the fifth line (line V) the R's (capital) represent places of maximum rarefaction while r's (small) denote places of lower rarefaction. The C's (capital) and c's (small) have similar significances for places of compression. The N's denote places of normal pressure.

25. Wave-length, Period, Frequency, Velocity of Propagation and the Relation among them—These physical quantities with reference to the longitudinal wave can be defined in the same words as in the case of the transverse wave (see Arts. 15, 17, 18, 19 and 20).

The distance AB (fig. 5) over which the disturbance travels during one complete vibration of the source is called the *wave-length* λ of the longitudinal wave. The particles at A and B are evidently in the same phase of vibration; any other particle in this phase of vibration will be situated at a distance λ or its multiple from A or B. Thus as in the case of transverse wave, the *wave-length is the shortest distance between two particles in the same phase of vibration.* Further, the particles at A and D are in opposite phases of vibration. Hence, the shortest distance between two particles in opposite phases of vibration amounts to half the wave-length. Similarly, the distance CE amounts

to half the wave-length and the distance AC or CD is equal to one-quarter of the wave-length.

The distance CC' or EE' is the maximum displacement of a particle on the longitudinal wave. The distance is called the *amplitude* of the longitudinal wave. It will be found that the amplitude of the longitudinal wave, is proportional to CC'' of EE'', the amplitude of the conventional displacement curve.

If n, T and V represent the frequency, period and velocity of propagation of the longitudinal wave respectively, defined as in the case of transverse wave we have the analogous relation as in the case of transverse wave (Art 21.)

$$VT = \lambda$$

or since $$T = \frac{1}{n},$$

$$n\lambda = V$$

We have seen that C and E (fig. 5) are places of normal pressure, the intervening region between them is one of compression. Further, the distance between C and E amounts to $\frac{\lambda}{2}$. Thus the length of a region of compression is $\frac{\lambda}{2}$ and similarly, the length of a region of rarefaction beside it is $\frac{\lambda}{2}$. Hence, a *longitudinal wave comprises one compressed region and one rarefied region.*

26. Wave-front, Ray : Plane and Spherical Waves—

A wave-front is a line or a surface on which the various particles are in the same phase of vibration.

If waves are generated on still water surface by producing periodic disturbance at a point on it, the particles of

water on a circle on the surface of water having the point as centre, will be in the same phase of vibration. Thus the wave-front consists of circles on the water surface having the point of disturbance as the centre.

If longitudinal waves are created in the air by the alternate compression and dilatation of a small sphere, the wave-front consists of spherical surface concentric with the small sphere. The waves thus produced are *spherical waves*. At a great distance from the source a spherical wave will have a large radius and a small portion of the wave surface may be regarded as sensibly plane and is termed a *plane wave-front*.

The direction of motion of a wave anywhere is along the perpendicular to the wave-front. If a line is drawn so as to indicate everywhere the direction of motion of the wave, it is called a *ray*.

Example

Carefully explain what goes on in a sounding body and the atmosphere in its neighbourhood. What do you understand by the wave-length of sound? *(C U. 1912)*

CHAPTER IV

PRINCIPLE OF SUPERPOSITION : PROGRESSIVE AND STATIONARY WAVES

27. Principle of Superposition—The principle of superposition was first studied by Huyghens in connection with light waves. The principle is also true for sound waves. It may be stated as,—"The resultant displacement, velocity, acceleration, state of compression or rarefaction due to a number of waves at any point in a medium is the *vector sum* of the corresponding quantities due to the separate waves."

From a physical standpoint, the application of the principle of superposition is a necessity. For, there cannot be two or more wave-systems simultaneously in the same part of the medium, as the same element of the medium cannot have two displacements or velocities at the same instant of time. The principle of superposition provides a ready method for calculating the resultant displacement, velocity, etc., at any point of the medium due to a num of waves. To apply the principle for finding the resultant displacement, velocity, etc., at a point due to a number of waves, first find the displacement, velocity, etc., produced at the point by each wave independently of the remaining waves. The resultant displacement, velocity, etc., will be obtained by combining the component quantities. Many observed phenomena based upon it test the truth of the principle. The phenomenon of interference, referred to in Art. 12, is a direct outcome of the principle of superposition.

The principle is valid so long as the displacements, accelerations, etc., due to the separate waves are not large and can be expressed as linear functions of the forces producing them. We shall return to a more detailed study of its limitations in Chapter XIII.*

It may be noted that the principle is applicable in the case of vector quantities such as, displacement, velocity, acceleration etc., but not in the case of scalar quantities such as, energy, intensity, etc.

28. Progressive Wave—A progressive wave may be defined as a continuous transfer of a particular state from one part of a medium to another, by similar movements performed successively by the consecutive particles of the medium.

The propagation of sound from one place to another ordinarily through air is an instance of *longitudinal progressive wave*. A particle of the medium vibrates to and fro *along* the line of propagation of the wave and transfers its state of displacement, acceleration, etc., to the next adjacent particle. Each particle is thus set to a similar vibratory motion executed along the line of propagation of the wave.

The propagation of light from the sun through ether is an instance of *transverse progressive wave*. A particle of ether vibrates *perpendicularly* to the line of propagation of the wave and transfers its state of vibration to the next particle. Each particle of ether is thus set to a similar vibratory motion executed perpendicularly to the line of propagation of the wave.

*See Art. 121.

On a progressive wave the various properties of any particle go through the *same* cycle of changes. Thus a particle on the wave occupies its undisplaced position at a particular instant of time. At another instant of time, it is displaced to the fullest extent. Similar is the change in displacement of any other particle. Thus the displacement of any particle goes through the *same* cycle of changes. Similarly, the velocity, acceleration, kinetic energy, potential energy, etc., of any particle go through the *same* cycle of changes.

29. Analytical Treatment of Progressive Waves—A particle on a progressive wave of the simple harmonic type executes simple harmonic vibration. Let a progressive wave travel along the positive direction of x and let y represent the displacement of a particle at a time t. For a longitudinal wave, y is evidently along the direction of x, and for a transverse wave, y is normal to the direction of x. The equation of motion of the vibrating particle can be put down as

$$y = a \cos \omega t, \quad \text{or} \quad y = a \sin \omega t^*$$

where a represents the amplitude of vibration of the particle or the amplitude of the wave and $\omega = 2\pi n$†, where n is the frequency of vibration of the particle or the frequency of the wave and ωt is the phase of the vibrating particle.

A particle vibrating at a distance x from the above will be executing similar vibrations but its phase is retarded. To calculate its retardation in phase, we see that the phase is retarded by 2π after the wave-length λ.†† Therefore the

* See 'Text-Book of General Physics' by the author, Art. 50.

† See 'Text-Book of General Physics' by the author, Art. 40.

†† See foot-note on page 16.

phase retardation of the particle vibrating at a distance x from the former amount to $\frac{2\pi}{\lambda}x$. Hence, its phase of vibration at the time under consideration amounts to $\omega t-\frac{2\pi}{\lambda}x$. *The displacement equation of the particle at the time t is thus given by

$$y=a\cos\left(\omega t-\frac{2\pi}{\lambda}x\right)$$

Since $\lambda=VT$, where V is the velocity of propagation of the wave and T its period, and $\frac{2\pi}{T}=\omega$, we have $\frac{2\pi}{\lambda}=\frac{2\pi}{VT}=\frac{V}{\omega}$ and the above equation can be put down as

$$y=a\cos\left(\omega t-\frac{\omega x}{V}\right)$$

$$\text{or } y=a\cos\omega\left(t-\frac{x}{V}\right) \quad \ldots(1)$$

Replacing ω by $\frac{2\pi}{T}$, equation (1) can be put down into the form

$$y=a\cos 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right) \quad \ldots (2)$$

Putting $\frac{\lambda}{T}=V$ in equation (2), it can be written as

$$y=a\cos\frac{2\pi}{\lambda}(Vt-x) \quad \ldots (3)$$

Any of the equations (1), (2) and (3) gives the displacement of a particle on a progressive wave and is the equation to the progressive wave.

* See 'Theory of Sound', Vol, I, Rayleigh.

If the vibratory motion of a particle is represented by $y=a\sin\omega t$, the equation to the progressive wave similarly becomes $y=a\sin\omega\left(t-\frac{x}{V}\right)$ and the equations (2) and (3) correspondingly take the forms $y=a\sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)$ and $y=a\sin\frac{2\pi}{\lambda}(Vt-x)$.

See 'A Text-Book of Sound', Barton.

30. Velocity of a Particle in the Progressive Wave—The velocity of a particle, at a place defined by the co-ordinate x and at a time t is obtained by differentiating its displacement with regard to time.

The displacement found in equation (1) of the previous article, is given by

$$y=a\cos\omega\left(t-\frac{x}{V}\right)$$

$\therefore$ the velocity $\frac{dy}{dt}$ of the particle

$$=-a\omega\sin\omega\left(t-\frac{x}{V}\right) \quad \ldots \quad (1)$$

The velocity of the particle, which depends both upon the time and place, should not be confused with the velocity of the wave V, which is a constant quantity depending upon the properties of the medium only.

Differentiating y with regard to x, we get $\frac{dy}{dx}$ which measures the slope of the displacement curve.

$$\frac{dy}{dx}=\frac{a\omega}{V}\sin\omega\left(t-\frac{x}{V}\right) \quad \ldots \quad (2)$$

Comparing (1) and (2) we find.

$$\frac{dy}{dt}=-V\frac{dy}{dx}$$

or the velocity of the particle $=-V\times$ *slope of the displacement curve.*

31. Acceleration of a Particle in the Progressive Wave—The acceleration of a particle at a time t and at a place defined by x is obtained by differentiating the velocity of the particle with regard to time. The velocity found in equation (1) of the last article is given by.

$$\frac{dy}{dt}=-a\omega\sin\omega\left(t-\frac{x}{V}\right).$$

$\therefore$ the acceleration $\frac{d^2y}{dt^2}$ of the particle

$$=-a\omega^2\cos\omega\left(t-\frac{x}{V}\right) \quad \ldots \quad \ldots \quad (1)$$

Differentiating equation (2) of the last article with regard to x,

$$\frac{d^2y}{dx^2} = -\frac{a\omega^2}{V^2}\cos\omega\left(t - \frac{x}{V}\right) \quad \dots \quad \dots \quad (2)$$

Comparing (1) and (2) we find,

$$\frac{d^2y}{dt^2} = V^2 \times \frac{d^2y}{dx^2}$$

or *the acceleration of the particle* $= V^2 \times$ *the rate of change of slope of the displacement curve.*

32. Energy of Sound Waves.—The kinetic energy per unit volume of the medium transmitting the progressive waves will be given by

$$K.\ E. = \tfrac{1}{2}\rho\left(\frac{dy}{dt}\right)^2$$

where ρ = mass of unit volume or the density of the medium and $\frac{dy}{dt}$ = particle velocity

$$K.\ E. = \tfrac{1}{2}\rho a^2\omega^2\sin^2\omega\left(t - \frac{x}{V}\right) \quad \dots \quad \dots \quad \text{(art. 30).}$$

But the sum of kinetic and potential energies is a constant, and when one kind of energy is a maximum the other kind of energy is zero*. Hence total energy per unit volume of the medium is given by the maximum kinetic or potential energy per unit volume. As the maximum value of $\sin^2\omega\left(t - \frac{x}{V}\right) = 1$,

the total energy per unit volume of the medium $= \tfrac{1}{2}\rho a^2\omega^2$. This is known as *energy density* of the medium.

To calculate the rate of flow of energy through the medium let us first of all find the energy content of a portion of the medium of which the cross-section perpendicular to

* See Author's General physics arts. 47, 48 and 49.

the direction of wave propagation is unity and of which the length is equal to λ, the wave-length of the waves. This energy is evidently the energy in a volume λ of the medium and is known as the energy per wave-length of the medium.

Total energy per wave-length of the medium $= \frac{1}{2}\rho a^2\omega^2\lambda$. This energy content per wave-length of the medium will be transmitted through unit area of the medium perpendicular to the direction of propagation of the wave in time T where T is the period of the wave; because the wave is transmitted through a distance λ in time T (art. 25). Hence the rate of flow of energy per unit area of the wave-front (art. 26).

$$= \tfrac{1}{2}\rho a^2\omega^2\lambda/T = \tfrac{1}{2}\rho a^2\omega^2 V.$$

This quantity is called the *energy current* of the medium and is regarded as the *intensity* of the sound.

Since $\omega = \dfrac{2\pi}{T} = 2\pi n$ where $n =$ frequency of the wave, the energy current $= 2\pi^2\rho a^2 n^2 V$.

Thus the intensity of the sound wave is directly proportional to the density of the medium and the wave velocity, it is also proportional to the square of the amplitude and square of the frequency of the wave.

33. **Stationary Waves**—Let us now study the effect of superposition of two sets of progressive waves having the same *amplitude* and *period* but travelling in *opposite* directions with the *same* velocity. This is a case which very often occurs with sound waves. For example, the longitudinal waves travelling from one end of an organ pipe through it get reflected at the other end (see Art. 139) and travel back. Thus within the organ pipe there are two sets

of progressive waves having the same amplitude and period but travelling in opposite directions with the same velocity.

We shall presently see, both graphically and analytically, that the result of superposition of the two sets of progressive waves is the formation of a system of waves alternately waxing and waning (expanding and shrinking), but without progression in either direction. These waves are called *stationary waves*. The two sets of longitudinal waves—one direct and the other reflected, in an organ pipe, as stated before, give rise to these stationary waves within the organ pipe. The waves are called stationary because they are confined in a certain region and non-progressive in character. The stationary waves formed within an organ pipe are confined within the organ pipe itself, and are of longitudinal character.

Similarly, when the string stretched on a sonometer between two bridges is plucked, the transverse waves thus set up travel along the string and are reflected from the bridges. Thus within the string there are two sets of progressive waves having the same amplitude and period but travelling in opposite directions with the same velocity. These give rise to the stationary waves of transverse character confined within the sonometer string.

The special features of stationary waves as distinct from those of progressive waves will be revealed from a pursuit of the graphical or analytical method of their formation.

The stationary waves have found application in the measurement of the depth of ocean in the American System. Lord Rayleigh used the stationary waves to

See Art. 69.

determine the frequency of a source emitting super-sonic waves.*

34. Formation of Stationary Waves—

(a) *Graphical Method.*

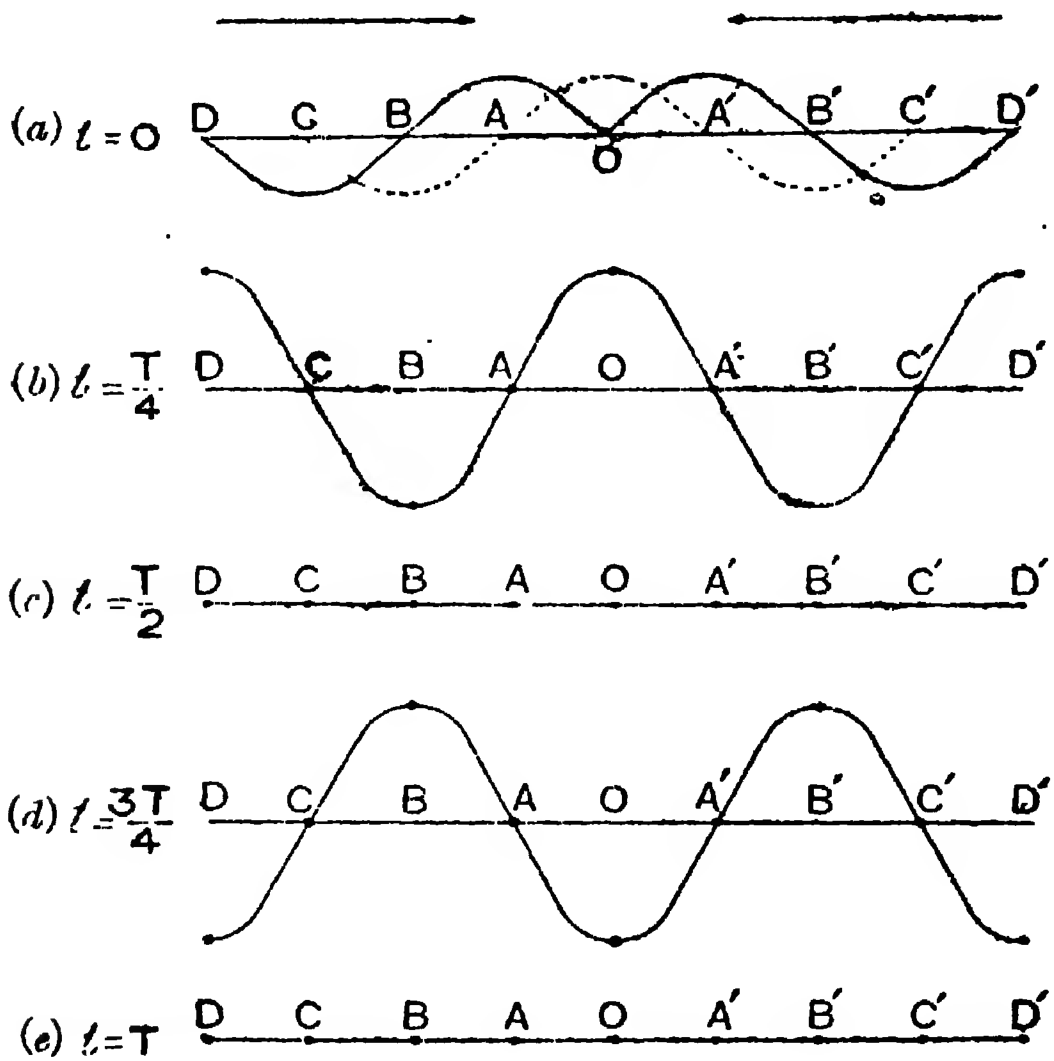
The graphical method of studying the formation of stationary waves is shown in fig. 7. The two wave DBO and $D'B'O$ represent the two sets of progressive wave trains having the same amplitude and period travelling along the same line in opposite directions shown by arrow-heads over them. Since the waves have the same period and velocity of propagation, their wave-lengths are the same (as $VT = \lambda$ Arts. 21 and 25). The waves meet each other at O shown in fig. 7 (a) at a particular instant of time which may be called the zero instant of time. Let us study the displacements of various particles of the line DOD' as time passes.

After the lapse of one-quarter of the periodic time of the wave, each wave advances a distance $\frac{\lambda}{4}$, so that each wave may be supposed to be bodily displaced through a distance $\frac{\lambda}{4}$ in the direction of its propagation as shown by the dotted lines in fig. 7 (a). The displacement of a particle of the medium situated on the line of propagation of the wave can be calculated by observing its displacement due to each wave and then by compounding its two displacements from the principle of superposition. Thus after the lapse of one-quarter of the periodic time the displacement of the particle at O amounts to a due to each

* See 'Theory of Sound', Rayleigh, vol, 2, p. 403.

Rayleigh's application has been improved to a greater precision by Pierce.

wave, where a represents the amplitude of either wave. Since both the displacements are directed upwards, the resultant displacement of the particle at O after one-



quarter of the periodic time amounts to $2a$ directed upwards.

The resultant displacement of the particle at O and at any other point can more conveniently be calculated by reference to fig. 7 (a) without drawing the dotted curves as follows :—In one-quarter of the periodic time each wave travels a distance $\frac{\lambda}{4}$ in its direction of propagation.

Hence after a time $\frac{T}{4}$, the particle at O gets the displacement of the particle at A due to the rightward-moving wave and that of the particle at A' due to the leftward-moving wave. Either of these displacements being a upwards, the resultant displacement of the particle at O is $2a$ upwards. This resultant displacement of the particle at O after a time $\frac{T}{4}$ is shown by a dot above the point O in fig. 7 (b). The displacement of a particle at A due to the rightward-moving wave is that of the particle at B, while that due to the leftward moving wave is that of the particle at O. Since both these displacements are zero, the resultant displacement of a particle at A is also zero. This is marked by a dot in fig. 7 (b). The displacement of a particle at B after a time $\frac{T}{4}$ due to the rightward-moving wave will be that of C, *i. e.* a downwards. Its displacement due to the leftward-moving wave will be that of A due to the *same* (leftward-moving) wave which can be observed if the wave curve is produced in imagination up to the point A. This latter displacement is also a downwards. Hence the resultant displacement of the particle at B after a time $\frac{T}{4}$ is $2a$ downwards as shown by a dot in fig. 7(b). The resultant displacemants of various particles at distances $\frac{\lambda}{4}$ are calculated in the above way and their displaced positions after the time $\frac{T}{4}$ are marked by dots in fig. 7(b). The wavy line joining these dots represents the form of the resulting wave after a time $\frac{T}{4}$.

To obtain the form of the resulting wave after the lapse of one-half of the periodic time, we see, that in this time each wave travels a distance $\frac{\lambda}{2}$ in its direction of propagation. Thus in fig. 7. (*a*) the particle at *O* will have the displacement of the particle at *B* due to the rightward-moving wave and also the displacement of the particle at *B'* due to the leftward-moving wave after the lapse of one-half of the periodic time. Both these displacements being zero, the resultant displacement of the particle at *O* is zero also. Similarly, the particle at *A* will have the displacement of the particle at *C*, *i. e.* *a* downwards due to the rightward-moving wave, and the displacement of the particle at *A'*, *i.e.* *a* upwards due to the leftward-moving wave. The resultant displacement of the particle at *A* is thus zero. It can be shown in a similar way that the displacements of all particles are zero also. The form of the resulting wave is thus a straight line as shown in fig. 7(*c*).

To study the form of the wave after the lapse of three-quarters of the periodic time, we see that each wave during this time travels a distance $\frac{3\lambda}{4}$ in its direction of propagation. Hence in fig. 7(*a*) the particle at *O* will have the displacement of the particle at *C*, *i. e.* *a* downwards due to the rightward-moving wave and the displacement of the particle at *C'*, *i. e.* *a* downwards due to the leftward-moving wave. The resultant displacement of the particle at *O* is thus $2a$ downwards. Calculating the displacement of the particles at *A*, *B*, *C*, *A'*, *B'*, *C'*, etc., we see that the form of the resulting wave is the reversal of fig. 7(*b*). The form of the wave is shown in fig. 7(*d*).

The form of the wave after the lapse of the periodic time T can be studied in a similar manner. The fig. 7 (e) shows the wave which is similar to fig. 7(c)

With further lapse of time the form of the wave curve will be the repetitions of the curves (b). (c) (d) and (e).

We see from the figures 7(b) to 7(e) that the displacement of the particle at certain points A, C, A', C', etc., on the wave is zero throughout the time. These points are called **nodes** or places of no displacement. On the other hand, at certain points O, B, D, B', D' etc., the particles of the medium have the maximum displacements $\pm 2a$. These points are called **antinodes** or places of maximum displacement. The maximum displacement at any other point lies between 0 and $\pm 2a$. Hence the positions of nodes and antinodes are *permanently* fixed.

It will be further observed that the amplitude of a stationary wave is double the amplitude of either of the progressive waves but the wave-length of the stationary was is the same as that of the either progressive wave.

(b) *Analytical Method*--

The equation to a progressive wave found in equation 1 of Art. 29 is given by

$$y_1 = a \cos \omega \left(t - \frac{x}{V} \right) \quad \dots \quad \dots \quad (1)$$

where y_1 is the displacement of a particle due to the wave moving in the positive direction of x.

The equation to a progressive wave moving in the negative direction of x and having the same amplitude, period

and velocity of propagation can be put down from equation (1) with a negative sign for V, *viz.*

$$y_2 = a \cos \omega\left(t + \frac{x}{V}\right) \quad \ldots \quad \ldots \quad (2)$$

where y_2 is the displacement of the same particle due to an identical wave moving in the negative direction of x. The resultant displacement y of the particle due to both the waves, will, by the principle of superposition, be given as*

$$y = y_1 + y_2$$

$$\therefore \quad y = a \cos \omega\left(t - \frac{x}{V}\right) + a \cos \omega\left(t + \frac{x}{V}\right)$$

$$\text{or} \quad y = 2a \cos \omega t \cos \frac{\omega x}{V} \quad \ldots \quad \ldots \quad (3)$$

This represents the equation to a stationary wave. It can be put into different forms similar to those in which the equation to a progressive wave is put in Art. 29.

Replacing ω by $\frac{2\pi}{T}$ and putting $VT = \lambda$, the equation (3) can be put into the form

$$y = 2a \cos \frac{2\pi t}{T} \cos \frac{2\pi x}{\lambda} \quad \ldots \quad \ldots \quad (4)$$

If $x = \frac{\lambda}{4}, \frac{3\lambda}{4} \ldots\ldots \frac{(2n+1)\lambda}{4} \ldots$, *i. e.* any odd multiple of $\frac{\lambda}{4}$, the equation (4) shows that $y = 0$ for all values of t.

*See Theory of Sound, Rayleigh, vol. 1, page 227.

If the equations of the component progressive waves be given as $y_1 = a \sin \omega\left(t - \frac{x}{V}\right)$ and $y_2 = a \sin \omega\left(t + \frac{x}{V}\right)$, the equation (3) to the stationary wave becomes $y = 2a \sin \omega t \cos \frac{\omega x}{V}$. See 'A Text-Book of Sound,' Barton, page 68.

These correspond to the nodes found to occur at space intervals of $\frac{\lambda}{2}$ in the graphical method.

If $x = 0, \frac{\lambda}{2}, \lambda \ldots \frac{(2n)\lambda}{4} \ldots$ *i.e.* any even multiple of $\frac{\lambda}{4}$,

$$y = \pm 2a \cos \frac{2\pi t}{T}$$

These correspond to the antinodes, the maximum magnitude of the displacement or the amplitude being $\pm 2a$ which is double the amplitude of the either progressive wave.

35. Distinction between a Progressive and a Stationary Wave—The difference between a progressive and a stationary wave can be summarised as follows :—

(1) A progressive wave advances onwards but a stationary wave is confined in the region where it is formed. It only shrinks to a straight line by proportional diminution of all its ordinates. It then expands proportionally, all the ordinates being now reversed in sign. It shrinks again and so forth.

(2) In a progressive wave, the various properties (displacement, velocity, acceleration, kinetic and potential energies, pressure-variation, etc.) of any particle pass through the *same* cycle of changes. In a stationary wave all the particles have not the *same* cycle of changes in their properties. At some points the particles are not displaced throughout the time, these are called the nodes ; at some other points the particles have amplitudes greater than those of any other particles, these are called the antinodes.

(3) In a progressive wave all the particles have the same amplitude but their phases are different. In a station-

ary wave all the particles between two successive nodes have the same phase but their amplitudes are different. At each node the phase changes by π, *i. e.* the particles on the opposite sides of a node move in opposite directions.

36. Velocity of a Particle in the Stationary Wave.—The velocity of a particle at a place defined by x, at the time t, is obtained by differentiating the expression for displacement with regard to time.

The expression for displacement obtained in Art. 33 equation (3) is given by

$$y = 2a \cos \omega t \cos \frac{\omega x}{V}$$

$\therefore$ The particle velocity $\dfrac{dy}{dt}$

$$= -2a\omega \sin \omega t \cos \frac{\omega x}{V}$$

37. Acceleration of a Particle in the Stationary Wave.—The acceleration of a particle at a place defined by x, at a time t, is obtained by differentiating the expression for velocity of the particle with regard to time. The velocity of the particle obtained in the previous article is given by

$$\frac{dy}{dt} = -2a\omega \sin \omega t \cos \frac{\omega x}{V}$$

$\therefore$ the particle acceleration $\dfrac{d^2 y}{dt^2}$

$$= -2a\omega^2 \cos \omega t \cos \frac{\omega x}{V}$$

$$= -\omega^2 y$$

Examples

1. Distinguish clearly between stationary waves and progressive waves.

Explain, graphically or mathematically, the formation of nodes and antinodes, when two trains of exactly similar waves travel along the same line from opposite directions. *(C. U. 1918)*

2. Give the theory of formation of stationary waves in air and in other elastic media, and illustrate your answer by considering the different types of stationary waves with which you are familiar in acoustics. *(C. U. 1920)*

3. State the chief characteristics of stationary waves showing particularly how they differ from progressive waves. *(C. U. 1936)*

4. Clearly distinguish between progressive and stationary waves.

A wave is propagating along a row of particles from a source having S. H. M. Find an expression for the displacement of y of a particle at a distance x from the source at time t. Hence show that every particle successively attains all the phases that simultaneously exist in the row of particles. *(C. U. 1937)*

5. Explain analytically the formation of stationary waves and show how they differ from progressive waves. *(C. U. 1938)*

6. Explain the formation of stationary waves when two trains of exactly similar waves are travelling along the same string from opposite directions. Obtain an expression for the distance between two nodes and for that between a node and an anti-node.

(C. U. 1939)

CHAPTER V

FREE AND FORCED VIBRATION : RESONANCE

5 56 **38. Free Vibration.**—When a body which is capable of vibrations or a system of bodies connected together to have a single period of vibration, is vibrating, its vibrations are called *free* if it is not subjected to any force impressed upon it externally. When a prong of a tuning fork is struck, the prongs vibrate due to *elastic* forces called into play. If the tuning fork is not subjected to any externally impressed force but simply disturbed so as to vibrate and then left to itself, the vibrations of the tuning fork are called *free* vibrations.

It should be understood that although 'free vibrations' are uninfluenced by externally impressed forces, still they are not unresisted from within. In the above instance of the free vibrations of a tuning fork, the different layers of the prong are moving relatively to one another during the vibratory motion of the fork. Therefore a force is called into play due to *viscosity* or *internal friction** of the material of the tuning fork. This internal friction mainly as well as the resistance offered by the air in which the fork vibrates is responsible for bringing its vibrations to ultimate subsidence. Vibrations, thus hampered from within, are also called *resisted* or *damped* vibrations.

If all the forces of frictional character are absent which is an ideal case, never realised but only approximated to in practice, the vibrations of the body or the system are called *natural* vibrations.

*See 'A Text-book of General Physics' by the author, Art. 126.

39. Analytical Treatment of Resisted Oscillations—The forces acting on a body executing damped vibrations are twofold—(1) A force called into play when it is displaced from its undisturbed position. This is the restoring force on the body directed opposite to its displacement and proportional to the displacement of the body in magnitude*: (2) A retarding force observed in the last article which to a first approximation is proportional to the velocity of the body. Thus if x represent the displacement of the body at a time t, the expression for force on the body is given by

$$m\frac{d^2x}{dt^2}=-a\frac{dx}{dt}-bx$$

where m is the mass of the body and a and b are constants.

The constant a is called the 'resistance constant' or the retarding force per unit velocity and b is called the 'spring factor' (stiffness factor) or the restoring force per unit displacement.

Dividing by m and transposing, the equation reduces to

$$\frac{d^2x}{dt^2}+\lambda\frac{dx}{dt}+\mu x=0 \qquad \ldots \qquad \ldots \qquad \ldots \qquad (1)$$

where $\lambda=\frac{a}{m}$ and $\mu=\frac{b}{m}$

To solve the equation (1) put

$$x=Ae^{kt}$$

$$\therefore \quad \frac{dx}{dt}=Ake^{kt}$$

and $\frac{d^2x}{dt^2}=Ak^2e^{kt}$

Putting these values of $\frac{d^2x}{dt^2}$, $\frac{dx}{dt}$ and x in equation (1),

$$Ak^2e^{kt}+\lambda Ake^{kt}+\mu Ae^{kt}=0$$

or $\quad k^2+\lambda k+\mu=0$

The above equation is quadratic in k and therefore admits of two values of k, which are

$$k=\frac{-\lambda\pm\sqrt{\lambda^2-4\mu}}{2}$$

* See 'A Text-Book of General Physics' by the author, Art. 46.

The value of the constant A in the solution assumed is left undetermined and is thus an arbitrary constant.

The constant k takes different values according as

$$\lambda^2 >, = \text{or} < 4\mu.$$

(1) If $\lambda^2 > 4\mu$, *i.e.* if the frictional forces are large, the values of k are real and negative, let them be represented by $-k_1$ and $-k_2$.

$\therefore$ The solution takes the form

$$x = A_1 e^{-k_1 t} \text{ or } x = A_2 e^{-k_2 t},$$

where A_1 and A_2 are arbitrary constants.

The general solution of equation (1) therefore is

$$x = A_1 e^{-k_1 t} + A_2 e^{-k_2 t}.$$

The solution shows that the motion is non-oscillatory. The body does not *vibrate* but with increase of time, its displacement subsides asymptotically to zero. It is called an *aperiodic*, *deadbeat* or *overdamped* motion.

(2) If $\lambda^2 < 4\mu$, *i. e.* if the frictional forces are small, the roots are imaginary having a real part.

Let the roots be represented by $-f \pm ig$

where $f = \dfrac{\lambda}{2}$ and $g = \dfrac{\sqrt{4\mu - \lambda^2}}{2}$

The general solution of the equation (1) is

$$\begin{aligned} x &= A_1 e^{(-f+ig)t} + A_2 e^{(-f-ig)t} \\ &= e^{-ft} \left(A_1 e^{igt} + A_2 e^{-igt}\right) \\ &= e^{-ft} \{A_1 (\cos gt + i \sin gt) + A_2 (\cos gt - i \sin gt)\} \\ &= e^{-ft} \{(A_1 + A_2) \cos gt + i (A_1 - A_2) \sin gt\} \end{aligned}$$

Choose two quantities C and θ such that

$$A_1 + A_2 = C \cos \theta \text{ and } i (A_1 - A_2) = C \sin \theta.$$

$$\therefore \quad x = e^{-ft} (C \cos gt \cos \theta + C \sin gt \sin \theta)$$

$$\text{or} \quad x = Ce^{-ft} \cos (gt - \theta).$$

The motion is thus *oscillatory*. This case is of frequent occurrence in acoustical phenomena. The damping has two effects upon the oscillatory motion of the body or the system. Firstly, the amplitude of the motion Ce^{-ft} decreases as time passes and the motion subsides *theoretically* after an infinite time. But *in practice* due to the inclusion of the exponential factor e^{-ft} the amplitude falls rapidly

to a low value within a short time after the motion is started. Secondly, we see from the equation the frequency of the damped motion

$$=\frac{g^*}{2\pi}=\frac{\sqrt{4\mu-\lambda^2}}{4\pi}=\frac{\sqrt{\mu-\lambda^2/4}}{2\pi}$$

while the natural frequency (frequency without damping)

$$=\frac{\sqrt{\mu}}{2\pi}\dagger$$

Thus the other effect of damping is to decrease the frequency or increase the period of oscillation, but the expressions above show that the effect of damping on the period of oscillation is of the *second order*; and to a *first approximation* we can say that the viscosity has no influence on the period but only on the amplitude.

(8) If $\lambda^2=4\mu$, $k=\frac{\lambda}{2}$

∴ the solution takes the form

$$x=Ae^{-\frac{\lambda}{2}t}$$

This is the transition case of '*critical damping*' when the motion just ceases to be *oscillatory* and becomes an *aperiodic* or *deadbeat* one.

40. Forced Vibration: Resonance—We have seen in the last two articles that owing to internal friction, a body set to oscillations and then left to itself gradually comes to rest by a progressive fall in amplitude. Therefore to maintain its vibrations some external impressed force must be applied to it. This fact has ample illustrations both mechanical and acoustical. Thus a pendulum once started and then left to itself finally comes to rest. To make the pendulum continue in its oscillation, a coiled spring is coupled to it in a clock. The sound emitted by a violin string quickly subsides, if its vibrations are not maintained by the continued action of the bow.

* See 'A Text-Book of General Physics' by the author, Art. 40

† See 'A Text-Book of General Physics' by the author, Art. 48.

Let us consider* the vibrations of an internally resisted body or a system of bodies having a single period of vibration, when subjected to an externally impressed force of the simple harmonic type. The period of vibration of the body or the system of bodies is its *natural period of vibration.* The externally impressed force which varies harmonically has a period of its own. If the natural period of vibration of the body or system is not the same as the period of the impressed force, the body or the system, when subjected to the periodic force, vibrates in an irregular manner at first due to the internal friction, etc. The effect of the internal friction, etc., becomes less and less important as time passes and ultimately the body or the system vibrates with a period as that of the externally impressed force. The vibration of the body or the system of bodies with a period as that of the impressed force, irrespective of its natural period of vibration, is called the *forced* vibration.

If it so happens that the period of the externally impressed force is the same as the natural period of vibration of the body or the system of bodies, the latter takes up its vibrations almost spontaneously and vibrates with the maximum amplitude. This particular case of forced vibration in which the period of the impressed force is the same as the natural period of vibration of the body or the system is called *resonance.*

It can be shown mathematically that if resonance occurs. *i. e.* if the period of the impressed force is the same as the natural period of vibration of the body or the system and the vibrations of the body are not resisted,—an ideal

* For mathematical treatment of the subject, see Art. 42.

case not realised in practice, the amplitude of vibration of the body or the system is infinitely large.

41. Illustrations of Forced Vibration and Resonance —(1) A *mechanical* illustration of forced vibration and resonance is shown in fig. 8. Three simple pendulums *A*, *B* and *C* are suspended from an india-rubber cord *DE* stretched between two rigid uprights of a massive rectangular frame-work. The pendulums *A* and *B* have same length while the length of *C* is different from that of *A* or *B*. Now let the pendulum *B* be set to oscillations. The vibrations of *B* provide a periodic force to the elastic support *DE* which is thereby set to forced transverse vibrations with the same period as that of *B*. The vibrations of the cord supply periodic forces to the other pendulums at their points of suspension.

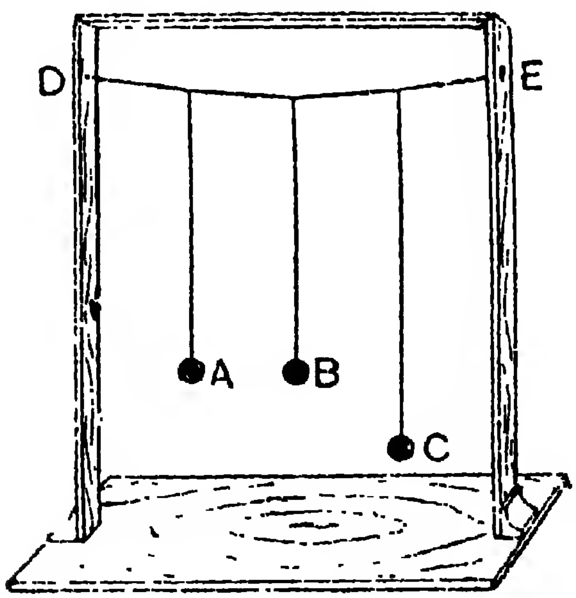


FIG. 8

It will be found that the pendulum *A* which has the same length and consequently the same period as that of *B*, takes up its vibrations very quickly. The pendulum *C* which has a different length and consequently a period of vibration different from that of *B*, shows at first an irregular motion but after a short time it will be found to vibrate in the same period as that of *B*. The amplitude of vibration of *A* will be found to be large compared to that of *C*. The vibrations of *C* are forced vibrations while the vibrations of *A* are resonant vibrations.

(2) The mounting of a tuning fork on its resonant box is an illustration of *acoustical* resonant. If the stem of a vibrating fork is pressed against a table, the table is set to forced vibration. The vibrations of the table add to the intensity of the sound emitted by the tuning fork. In order to obtain a largely intense sound the tuning fork is mounted on a hollow wooden box, the size of which is such that the air column within it has the same period of vibration as the tuning fork. The vibrations of the tuning fork set the enclosed air column to resonant vibrations with a large amplitude and a very intense sound is produced thereby.

(3) The principle of resonance is applied in receiving the *electromagnetic* waves in the wireless telegraphy. The 'oscillating circuits' of the sending and receiving stations have the same period. Instead of applying the term resonance, which is commonly used in acoustics, the oscillating circuits are said be '*in tune.*' The proper *tuning* of the two oscillating circuits is responsible for the transmission of messages.

42. Analytical Treatment of forced Vibration and Resonance. (a) *Forced Vibration.*—We have seen in Art. 89 that in absence of any externally impressed force, there are two opposing forces on the motion of a vibrating body—one is the retarding force proportional to its velocity and the other is the restoring force proportional to its displacement. If an impressed force of harmonic character acts upon it, the expression for the force acting upon the body at a time t when its displacement is x can be put down as

$$m\frac{d^2x}{dt^2} = -a\frac{dx}{dt} - bx + F\cos pt$$

Dividing both sides by m and re-arranging the terms, the equation is reduced to

$$\frac{d^2x}{dt^2} + \lambda\frac{dx}{dt} + \mu x = f\cos pt \quad \ldots \quad \ldots \quad (1)$$

$$\text{where } \lambda = \frac{a}{m},\ \mu = \frac{b}{m} \text{ and } f = \frac{F}{m}$$

To solve the equation try a particular solution

$$x = A\cos(pt - \phi)$$

$$\therefore \quad \frac{dx}{dt} = -Ap\sin(pt - \phi)$$

$$\text{and } \frac{d^2x}{dt^2} = -Ap^2\cos(pt - \phi)$$

Putting down the values of $\frac{d^2x}{dt^2}$, $\frac{dx}{dt}$ and x in equation (1)

$$-Ap^2\cos(pt - \phi) - \lambda Ap\sin(pt - \phi) + \mu A\cos(pt - \phi) = f\cos pt.$$

Since the above equation holds good for all time, the time can be suitably chosen such that $\sin pt = 0$, when both the sides include terms containing $\cos pt$ only. We can thus equate the coefficients of $\cos pt$. Similarly, the time can be suitably chosen to make $\cos pt = 0$, so that both sides contain products of $\sin pt$ only. We can thus equate the coefficients of $\sin pt$.

Equating the coefficients of $\cos pt$

$$-Ap^2\cos\phi + \lambda Ap\sin\phi + \mu A\cos\phi = f$$

$$\text{or } A(\mu - p^2)\cos\phi + \lambda Ap\sin\phi = f \quad \dots \quad \dots \quad (2)$$

Equating the coefficients of $\sin pt$

$$-Ap^2\sin\phi - \lambda Ap\cos\phi + \mu A\sin\phi = 0$$

$$\text{or} \quad A(\mu - p^2)\sin\phi - \lambda Ap\cos\phi = 0 \quad \dots \quad \dots \quad (3)$$

Squaring (2) and (3) and adding together

$$A^2\{(\mu - p^2)^2 + \lambda^2 p^2\} = f^2$$

$$\because \quad A^2 = \frac{f^2}{(\mu - p^2)^2 + \lambda^2 p^2} \quad \dots \quad (4)$$

From relation (3), we have

$$\tan\phi = \frac{\lambda p}{\mu - p^2} \quad \dots \quad \dots \quad \dots \quad \dots \quad (5)$$

Equations (4) and (5) determine the values of A and ϕ in the solution.

The general solution of the equation (1), when the internal resistance is small, is the sum of the solution of Art. 89. (2) and of the above solution. Therefore the general solution of equation (1) is given by

$$x = Ce^{-ft}\cos(gt - \theta) + A\cos(pt - \phi).$$

The first term of the above solution represents what Rayleigh calls the *free* vibration of the body. 'It is that executed by the body or the system, when disturbed from equilibrium and then, *left to itself*.' The second term represents what he calls the *forced* vibration. 'It is the response of the body or the system to a force impressed upon it from without, and is maintained by the continued operation of that force. The amplitude of vibration is proportional to the amplitude of the force, and the period is the same as that of the force.' The inclusion of the first term in the above value of x shows that the vibrations of the body or the system are at first irregular. With increase of time the first term becomes less and less important and the body or the system finally vibrates with a period as that of the applied force. Equation (8) determines the difference in phases between the applied force and the vibration of the body.

(b) *Resonance*—The *natural* period of vibration of the body

$$=\frac{2\pi}{\sqrt{\mu}}\,*$$

The period of the applied force $=\frac{2\pi}{p}$ †

When these two periods are equal, $\mu=p^2$

From equation (4) we see, that this is the condition for the maximum value of the amplitude A. Thus resonance occurs when the period of the applied force is equal to the natural period of vibration of the body.

If $\mu=p^2$ and $\lambda=0$, $A=\infty$ from equation (4)

i. e. If resonance occurs and the vibrations of the body are not resisted, which is only theoretical, never realised in practice, the amplitude of vibration becomes infinitely large.

Examples

1. What is meant by reasonance? In what different ways can the sound emitted by a vibrating tuning fork be made audible at a

* See 'A Text-Book of General Physics' by the author, Art. 48.

† See 'A Text-Book of General Physics' by the author, Art. 40.

considerable distance? Show that the different means of producing this effect are not strictly analogous to one another. How can the vibration of the prongs of a tuning-fork be made visible at a distance and how can they be utilized to measure small intervals of time. (*C. U. 1910*)

2. Write a short essay on resonance, pointing out how the subject may be experimentally illustrated. (*C. U. 1933*)

3. Why does the faint sound from a tuning-fork becomes quite loud on placing the shank upon a table top? (*C. U. 1941*)

CHAPTER VI

VELOCITY OF LONGITUDINAL WAVES THROUGH GASEOUS MEDIUM

43. Let us now determine the speed of longitudinal waves through a gaseous medium, the following deduction is analogous to Professor Tait's method of finding the velocity of transverse waves along a stretched cord and Professor Barton calls it 'the simplest and most elegant of all elementary methods.'

Let the sound waves travel in the direction AB (fig. 9) through the medium with a velocity V. Consider a tube $ABCD$ of the medium having unit sectional area. For definiteness choose two points A and B fixed in space such that A is a region of normal pressure and B that of rarefaction at a particular instant of time.

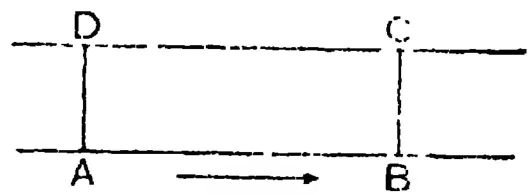


FIG. 9

Further, let us suppose that the medium is made to move in the direction opposite to that of sound waves with a velocity V, such that the sound waves in the region $ABCD$ cease to advance. This being so, the velocity of the particles of the medium, their states of displacement, the pressure and the density of the medium, though different at different sections of the region $ABCD$, retain their values throughout the time. Since the plane section at A is of normal pressure, the particles of the medium at this plane will have no velocity of their own (see Arts. 23, 24, fig. 6) but due to the opposing

motion of the medium a volume V will be passing out of the region $ABCD$ per second through the section AD. The particles of the medium at the plane section at B which is a region of rarefaction will have a velocity from right to left which combined with the velocity of the medium will cause a volume V', greater than V, to enter into the region $ABCD$ per second through the section BC.

But since the pressure and hence density of any point within the region $ABCD$ remains unaltered, the mass of the medium contained in the region remains the same. Hence the mass of the medium leaving the region per second through AD is the same as the mass of the medium entering the region through BC per second. Thus if ρ and ρ' be the densities of the medium at A and B respectively, we have the so-called equation of continuity.

$$V\rho = V'\rho' \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Again, although the masses of the medium crossing AD and BC per second have the same value, still because of their unequal velocities, they possess different momenta. The momentum gained by the region $ABCD$ through BC per second is $V'\rho'.V'$ or $V'^2\rho'$ while that lost through AD per second is $V\rho.\,V$ or $V^2\rho$. Hence the net gain of momentum per second in the region $= V'^2\rho' - V^2\rho$.

By Newton's second law of motion this must be equal to the external force on the layer of the medium in the region $ABCD$. Since the sectional area of the tube is unity, the external force is equal to the difference in pressures between its ends. Hence if P and P' be the pressures at A and B respectively,

$$P - P' = V'^2\rho' - V^2\rho \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Substituting the value of V' deduced from (1) in (2),

$$P-P' = \frac{V^2\rho^2}{\rho'} - V^2\rho$$

$$= V^2\rho\left(\frac{\rho}{\rho'}-1\right) = V^2\rho\frac{\rho-\rho'}{\rho'}$$

$$\therefore \quad V^2 = \frac{1}{\rho}\,\frac{P-P'}{\frac{\rho-\rho'}{\rho'}}$$

If u and u' represent the specific volumes at A and B respectively, $\rho = \frac{1}{u}$ and $\rho' = \frac{1}{u'}$.

$$\therefore \quad V^2 = \frac{1}{\rho}\,\frac{P-P'}{\frac{\frac{1}{u}-\frac{1}{u'}}{\frac{1}{u'}}} = \frac{1}{\rho}\,\frac{P-P'}{\frac{u'-u}{u}}$$

But $\dfrac{P-P'}{\frac{u'-u}{u}}$ measures the modulus of volume elasticity E of the medium.

Hence $V^2 = \dfrac{E}{\rho}$ or $V = \sqrt{\dfrac{E}{\rho}}$

44. The following treatment* for the velocity of propagation of longitudinal waves through a gaseous medium is instructive.

Consider a tube of the gaseous medium such as air, of unit sectional area and two plane sections A and B of the tube (fig. 10).

Let the positions of A and B be defined by their distances from some fixed point on the tube to the left of A. Let these respective distances be x and $x+\delta x$ and the

* This analytical theory of the plane waves of sound is due to Euler (1747) and Lagrange (1759)

positions of A and B be thus defined by the co-ordinates x and $x+\delta x$, respectively. The length of the slice AB is evidently δx and since the sectional area of the tube is unity, the volume of the slice is $1 \times \delta x$ or δx.

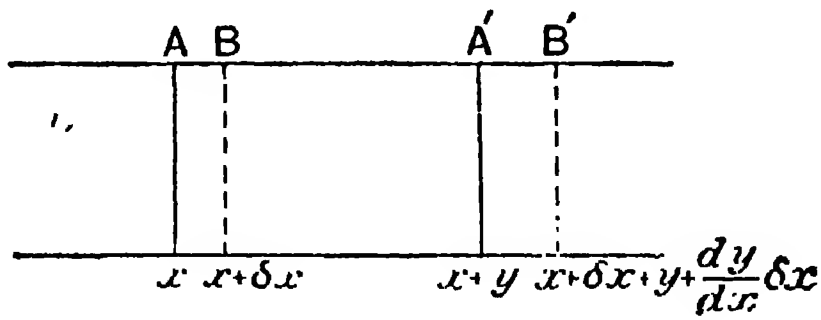


FIG. 10

Let the arrival of a longitudinal wave travelling in the direction of x, subsequently displace the plane section A through y. Since in a longitudinal wave the displacements of particles and direction of propagation of the wave are along the same line, the actual position of the plane section measured from the same fixed point will be given by the co-ordinate $x+y$. This is shown by A' in the figure As y measures the displacement in the direction of x, $\frac{dy}{dx}$ measures the rate of displacement with respect to *distance*, along the direction of x. Thus the change of displacement between the plane sections A and B,

$$=\frac{dy}{dx}\delta x$$

and the displacement of the plane section B

$$=y+\frac{dy}{dx}\delta x.$$

$\therefore$ the actual position of the plane section B

$$=x+\delta x+y+\frac{dy}{dx}\delta x.$$

This is shown by B' in the figure 10.

The length of the slice AB in the displaced position $A'B'$ is thus $\delta x+\frac{dy}{dx}\delta x$, and since the sectional area of the tube is unity, the volume of the slice in the displaced position is

$$\delta x+\frac{dy}{dx}\delta x$$

The change in volume of the slice is thus

$$\delta x+\frac{dy}{dx}\delta x-\delta x=\delta x\frac{dy}{dx}$$

and the fractional change in volume or the volume strain

$$=\frac{\frac{dy}{dx}\delta x}{\delta x}=\frac{dy}{dx}$$

If p represents the excess of pressure (above normal) over the plane section A the modulus of volume elasticity E of the medium* is given by

$$E=\frac{p}{-\frac{dy}{dx}}$$

(the negative sign is due to the fact that an increase of pressure causes a decrease in the volume and vice-versa. Hence p and $\frac{dy}{dx}$ are of opposite signs.)

Thus the excess of pressure (p) over the plane section A

$$=-E\frac{dy}{dx} \qquad \ldots \qquad \ldots \qquad (1)$$

The excess of pressure (above normal) over the plane section B

$$=-E\frac{dy}{dx}+\frac{d}{dx}\left(-E\frac{dy}{dx}\right)\delta x$$

$$=-E\frac{dy}{dx}-E\frac{d^2y}{dx^2}\delta x \qquad \ldots \qquad \ldots \qquad (2$$

* See 'A Text-Book of General Physics' by the author, Art.

From relations (1) and (2), the difference of pressures over the plane sections A and B

$$=E\frac{d^2y}{dx^2}\delta x$$

This is the moving force on the gas in the slice AB, as the sectional area of the tube is unity. The mass of the gas in motion $= 1 \times \delta x \times \rho$ where ρ is the density of the gas.

By Newton's second law of motion

$$E\frac{d^2y}{dx^2}\delta x = 1 \times \delta x \times \rho\frac{d^2y}{dt^2} \qquad \ldots \qquad \ldots \qquad (3)$$

as $\frac{d^2y}{dt^2}$ measures the acceleration of the motion.

$\therefore$ from (3),

$$\frac{d^2y}{dt^2} = \frac{E}{\rho}\cdot\frac{d^2y}{dx^2}$$ (2)A

If V^2 is put for $\frac{E}{\rho}$, the equation (D' Alembert's equation) reduces to

$$\frac{d^2y}{dt^2} = V^2\frac{d^2y}{dx^2} \qquad \ldots \qquad \ldots \qquad (4)$$

The general solution of the above equation (4) is

$$y = f(x - Vt) + F(x + Vt)^* \qquad \ldots \qquad \ldots \qquad (5)$$

where the functions f and F denote arbitrary functions

---

* The solution can be verified in the following way :—

Differentiating (5) twice with regard to x,

$$\frac{d^2y}{dx^2} = f''(x - Vt) + F''(x + Vt)$$

Differentiating (5) twice with regard to t.

$$\frac{d^2y}{dt^2} = V^2 f''(x - Vt) + V^2 F''(x + Vt)$$

$$\therefore \quad \frac{d^2y}{dt^2} = V^2\frac{d^2y}{dx^2}$$

The two terms in the solution (5) can be interpreted in the following way. The first term $y=f(x-Vt)$ is a particular solution of equation (4). This shows that if t increases by unity, y remains unchanged if x increases by V. In other words, the displacement which exists at a point x at the instant of time t, is found at a point $x+V$ after unit time. This is true if the wave travels unchanged through a distance V in the positive direction of x in unit time. Thus, this part of the solution represents a wave-form travelling in the direction of x-positive with a velocity V.

The second term $y=F(x+Vt)$, the other particular, solution of equation (4), shows that if t increases by unity, y remains unchanged if x changes by $-V$. In other words, the displacement which exists at a place x at the instant of time t is found at a place $x-V$ after unit time. This is true if the wave travels unchanged through a distance V in the negative direction of x in unit time. Thus this part of the solution represents a wave from travelling in direction of x-negative with a velocity V.

The velocity of propagation V is thus given by

$$V=\sqrt{\frac{E}{\rho}}$$

where E is the modulus of volume or bulk elasticity and ρ the density of the gaseous medium.

15. Newton's Value and Laplace's Correction—The expression for the velocity of propagation of longitudinal waves $V=\sqrt{E/\rho}$ in gases was first obtained by Newton. He assumed that during the propagation of longitudinal waves, the alternate compressions and rarefactions took place so slowly that the heat developed as a result of compression of a layer was *fully* dissipated away into the body

of the gas. Similarly, the cold produced in a rarefied layer was *fully* compensated for by the heat of the surrounding medium. There was thus no change of temperature during the propagation of the longitudinal waves and hence the value of E in the above expression should be deduced under *isothermal condition.* It can be shown that the modulus of bulk elasticity of a perfect gas under isothermal condition is equal to the pressure of the gas.

Let the pressure P of a gas be increased isothermally to $P+p$ and let its volume in consequence decrease from v to $\text{v}-v$; we have from Boyle's law

$$P\text{v}=(P+p)(\text{v}-v)=P\text{v}+p\text{v}-vP-pv$$

Neglecting pv which is a product of two small quantities, and therefore of higher order of smallness than any other term of the above expression, we have

$$P\text{v}=P\text{v}+p\text{v}-vP$$

$$\text{or} \quad vP=p\text{v}$$

$$\text{or} \quad P=\frac{p}{v/\text{v}}$$

Again $\dfrac{p}{v/p}$ measures the modulus of bulk elasticity E of the gas. Therefore $E=P$

Hence the formula for the velocity of longitudinal waves in a gas can be put down as

$$V=\sqrt{\frac{P}{\rho}}$$

Under normal conditions, the pressure of the atmosphere $=76$ cms. of mercury. In absolute measure this is

* For a treatment of this involving calculus, see 'A Text-Book of General Physics' by the author, Art. 118a.

$76 \times 13 \cdot 6 \times 981$ or $1 \cdot 014 \times 10^6$ dynes per sq. cm. approximately. The density of dry air under normal conditions $= \cdot 001293$ gms. per c. c. The velocity of sound in dry air at N. T. P. calculated from these data

$$= \sqrt{\frac{1 \cdot 014 \times 10^6}{\cdot 001293}} \text{ cms. per second}$$

$$= 280 \text{ metres per second nearly.}$$

Now the earliest experiments on the velocity of sound had shown a value 330 metres per second nearly, a value greater than the theoretical value by a little above one-sixth. Newton ascribed this discrepancy to the fact that the molecules of air were incompressible, so that the disturbance passed instantaneously from one end of a molecule to the other end of it but took time to pass through the inter-spaces. This assumption raised his theoretical value by about one-eighth. He further supposed that the water vapour present in the air took no part in the propagation. However, there was no basis of any of these suppositions.

The source of error was pointed out 120 years later by Laplace in 1817. He held that the alternate compressions and rarefactions took place so quickly that the heat developed in the compressed layer had no time to be dissipated into the body of the gas but remained *fully* lodged into the compressed layer. Similarly, the cold produced in the rarefied layer remained *wholly* in it. The condition is therefore *adiabatic*. It can be shown that the modulus of volume elasticity of a perfect gas under adiabatic condition is equal to $\gamma \times$ the pressure of the gas, where γ is the ratio between the specific heats of the gas at constant pressure and at constant volume.

Let the pressure P of a gas be increased adiabatically to $P+p$ and let its volume in consequence decrease from v to $\mathrm{v}-v$; we have for the adiabatic relation of gas.

$$P\mathrm{v}^{\gamma}=(P+p)(\mathrm{v}-v)^{\gamma}=(P+p)\mathrm{v}^{\gamma}\left(1-\frac{v}{\mathrm{v}}\right)^{\gamma}$$

$$=(P+p)\mathrm{v}^{\gamma}\left(1-\frac{\gamma v}{\mathrm{v}}\right) \text{ approximately ;}$$

since v is small compared to v, terms containing $\frac{v^2}{\mathrm{v}^2}$ and higher powers of $\frac{v}{\mathrm{v}}$ are small compared to terms containning v/v,

$$\therefore \quad P=(P+p)\left(1-\frac{\gamma v}{\mathrm{v}}\right)=P-P\frac{\gamma v}{\mathrm{v}}+p-\frac{\gamma pv}{\mathrm{v}}.$$

The term $\frac{\gamma pv}{\mathrm{v}}$ is small compared to any other term of the expression as it involves the product of two small quantities p and v.

$$\therefore \quad P=P-P\frac{\gamma v}{\mathrm{v}}+p \qquad \text{or } P\frac{\gamma v}{\mathrm{v}}=p$$

or $P\gamma=\dfrac{p}{v/\mathrm{v}}=E$, the modulus of bulk elasticity of the gas.*

Hence the formula for the velocity of longitudinal waves in a gas can be put down as

$$V=\sqrt{\frac{\gamma P}{\rho}}$$

The value of γ is 1·67 for a monatomic gas, 1·41 for air or any diatomic gas, 1·33 for a triatomic gas, etc. †

* For a treatment of this involving calculus, see the author's 'Text-Book of General Physics', Art. 117*b*.

† The value of γ is deduced from the kinetic theoryof gases. See he author's Text Book of General Physics,' Art. 166.

Laplace's assumption was put to test and was found to be in excellent agreement with experimental results which appear to justify Laplace's assumption.

It might be expected that the condition of the medium during the propagation of the longitudinal waves through it was neither isothermal nor adiabatic but intermediate between the two, so that the heat developed in a compressed layer partly passes away into the body of the gas and partly remains in the layer. Similarly, the cold in the rarefied layer is partly compensated for by the heat from the medium and the temperature of the layer drops partly.

This objection was refuted by Stokes, who showed theoretically that the condition must be either isothermal or adiabatic but not intermediate between the two.* With any intermediate state there will be so much dissipation of sonorous energy (conversion of sound energy into heat) that the sound waves will die away before they have travelled a short distance. Since there is no exprimental evidence of such quick dissipation of sound energy, the state will be either isothermal or adiabatic. The state is thus adiabatic supposed by Laplace and upheld by experimental agreement.

46. Effect of Pressure, Temperature, Moisture, etc. on the Velocity of sound—

(a) *Effect of Pressure*—If the temperature of a gas remains constant, a change of pressure does not influence the velocity of sound waves through it. This is due to the fact that with a change of pressure the density of the gas

* See Theory of sound, Rayleigh, Vol II, page 24 or the original paper of Stokes in Phil. Mag. 1. 305. 1851.

changes proportionately such that the ratio between pressure to density remains unaltered. Let V represent the velocity of sound in a gas when its pressure is P, its density ρ and the specific volume u, and let V' represent the velocity when the pressure is P', density ρ' and specific volume u'.

$$\therefore \quad V = \sqrt{\frac{\gamma P}{\rho}} \qquad \text{and } V' = \sqrt{\frac{\gamma P'}{\rho'}}$$

Again, from Boyle's law

$$Pu = P'u' \qquad \frac{P}{\rho} = \frac{P'}{\rho'}$$

Since $u = \dfrac{1}{\rho}$ and $u' = \dfrac{1}{\rho'}$, $\dfrac{P}{\rho} = \dfrac{P'}{\rho'}$

$$\therefore \quad V = V'$$

Or *the velocity is unchanged by any change of pressure.*

(b) *Effect of Temperature*—A change of temperature affects the density of a gas and hence the velocity of sound through it. Let V_0 and V_t represent the velocities of sound in a gaseous medium when the corresponding temperatures on the Centigrade scale are 0° and t°.

If the pressure has the same value P and the densities of the medium at temperatures $0^\circ C$ and $t^\circ C$ be ρ_o and ρ_t respectively,

$$V_o = \sqrt{\frac{\gamma P}{\rho_o}} \text{ and } V_t = \sqrt{\frac{\gamma P}{\rho_t}}$$

Dividing,

$$\frac{V_t}{V_o} = \sqrt{\frac{\rho_o}{\rho_t}} = \sqrt{\frac{\rho_t(1+at)}{\rho_t}}$$

(where a is the coefficient of expansion of the gas at constant pressure = ·00366 nearly)

$$\therefore \quad \frac{V_t}{V_o} = \sqrt{1+at} \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$= (1+at)^{\frac{1}{2}} = 1 + \frac{1}{2}at$ approximately,
(neglecting a^2t^2 etc compared to at)

$\therefore \quad V_t = V_0(1 + \frac{1}{2}at).$

This relation can be put in a different way :—

From (1) $\frac{V_t}{V_0} = \sqrt{1 + \frac{t}{273}}$

(as for a gas obeying Boyle's law $a = \frac{1}{273}$)

$$\therefore \quad \frac{V_t}{V_0} = \sqrt{\frac{273 + t}{273}} = \frac{\sqrt{T_t}}{\sqrt{T_0}}$$

where T_0 and T_t are temperatures on the absolute scale corresponding to O° and t° on the Centigrade scale.

$\therefore$ *the velocity of sound in a gas is directly proportional to the square root of absolute temperature of the gas.*

(c) *Effect of Moisture*—The presence of moisture in air lowers its density and therefore increases the velocity of sound in it. The higher the degree of saturation of the air, the greater will be the velocity of sound waves through it.

Let V_D represent the velocity of sound in dry air at a temperature t and pressure P and V_M the velocity in moist air at the same temperature and pressure.

If ρ_D and ρ_M are the densities of dry and moist air at the temperature and pressure, we have

$$V_D = \sqrt{\frac{\gamma P}{\rho_D}} \text{ and } V_M = \sqrt{\frac{\gamma P}{\rho_M}}$$

$$\therefore \quad \frac{V_D}{V_M} = \sqrt{\frac{\rho_M}{\rho_D}} \text{ or } V_D = V_M\sqrt{\frac{\rho_M}{\rho_D}} \qquad (1)$$

Let B be the height of the mercury barometer in cms. of which P is the equivalent pressure in absolute units

(dynes per sq. cm.) and f the pressure of aqueous vapour present in it in cms. of mercury. The partial pressures of dry air and aqueous vapour are therefore $B-f$ cms. and f cms. of mercury, respectively.

ρ_M = density of moist air at pressure B and temperature t of the atmosphere

= mass of 1 c.c. of moist air at pressure B and temperature t

= mass of 1 c.c. of dry air at pressure $(B-f)$ and temperature t + mass of 1 c.c. of water vapour at pressure f and temperature t

= mass of $\frac{B-f}{B}$ c.c. of dry air at a pressure B and temp. t + mass of $\frac{f}{B}$ c.c. of water vapour at pressure B and temp. t (By Boyle's law)

$$=\frac{B-f}{B}\rho_D+\frac{f}{B}\times \cdot 622\,\rho_D$$

(as the specific gravity of water vapour with regard to air at the same temp. and pressure is ·622)

$$\therefore \quad \rho_M=\frac{\rho_D}{B}(B-f+\cdot 622f)$$

$$=\frac{\rho_D}{B}(B-\cdot 378f)$$

$$\text{Or}\quad \frac{\rho_M}{\rho_D}=\frac{B-\cdot 378f}{B}$$

Substitute this value of $\frac{\rho_M}{\rho_D}$ in (1)

$$\therefore \quad V_D=V_M\sqrt{\frac{B-\cdot 378f}{B}}$$

(d) *Effect of Wind*—The velocity of sound in free air is affected by the wind. If the wind blows in the direction of propagation of sound, the observed velocity of sound

will be $V+w$ or $V-w$ according as the wind is with or against the sound, where V is the velocity of sound in still air and w, the velocity of wind. If the direction of the wind is different from that of the sound, the component of the wind velocity along the direction of sound propagation will affect the velocity of sound.

47. Velocity of Sound in a Mixture of Gases—A theoretical calculation of the velocity of sound in a mixture of several gases can be made from a knowledge of the partial pressures and densities of the constituent gases.

Let a mixture of pressure P and temperature t be made of several gases having partial pressures p_1, p_2, p_3, etc. in the mixture. If the densities of the constituent gases under a pressure P and temperature t be respectively ρ_1, ρ_2, ρ_3 etc., etc., it can be shown that the density ρ of the mixture is given by

$$P\rho = p_1\rho_1 + p_2\rho_2 + p_3\rho_3 + \ldots\ldots$$

$$\text{or } \rho = \frac{p_1\rho_1 + p_2\rho_2 + p_3\rho_3 + \ldots\ldots\ldots}{P} \quad \ldots \quad \ldots \quad (1)$$

Also if γ_1, γ_2, γ_3, *etc.* be the ratios of the specific heats

* ρ_1, ρ_2, ρ_3, etc. are the densities of the constituent gases under a pressure P. Since under isothermal conditions, the density of a gas varies *directly* as its pressure (Boyle's Law), the densities of the constituent gases under their partial pressures p_1, p_2, p_3, etc. will be

$$\frac{p_1\rho_1}{P}, \frac{p_2\rho_2}{P}, \frac{p_3\rho_3}{P}, \text{etc.}$$

The mass of the mixture will be the sum of the masses of the constituent gases. Hence, if v be the volume of the mixture, v is also the volume of each of the constituent gases and we have

$$v\rho = v\frac{p_1\rho_1}{P} + v\frac{p_2\rho_2}{P} + v\frac{p_3\rho_3}{P} + \ldots$$

$$\text{or} \quad \rho = \frac{p_1\rho_1 + p_2\rho_2 + p_3\rho_3 + \ldots\ldots}{P}$$

at constant pressure and at constant volume of the constituent gases, the same quantity γ for the mixture is given by

$$\frac{P}{\gamma-1}=\frac{p_1}{\gamma_1-1}+\frac{p_2}{\gamma_2-1}+\frac{p_3}{\gamma_3-1}+\ldots \qquad \ldots \qquad (2)^*$$

Equations (1) and (2) give the values of the density and ratio of the specific heats of the mixture in terms of those of the constituent gases. The value of the velocity of propagation of sound through the mixture can then be obtained from Laplace's formula.

$$V=\sqrt{\frac{\gamma P}{\rho}}$$

48. Experiment on the Velocity of Sound in Air—Experiments made to determine the velocity of sound in air

*Let the volume v of a gas at a particular temperature expand by dv at constant pressure due to unit rise in temperature. If M be the mass of the mixture, c_p and c_v its specific heats at constant pressure and at constant volume, we have taking v for the volume of the mixture, $M(c_p-c_v)=\frac{Pdv}{J}$, where J is the mechanical equivalent of heat, or $Mc_v\,(\gamma-1)=\frac{Pdv}{J}$ or $Mc_v=\frac{Pdv}{J(\gamma-1)}$

If m_1, m_2, etc. be the masses and c_1, c_2, etc. be the specific heats at constant volume of the constituent gases, we have analogous relations.

$$m_1c_1=\frac{p_1dv}{J(\gamma_1-1)} \qquad m_2c_2=\frac{p_2dv}{J(\gamma_2-1)}\text{, etc.}$$

Since the thermal capacity of the mixture = the sum of the thermal capacities of the constituent gases,

$$Mc_v=m_1c_1+m_2c_2+\ldots\ldots\ldots\ldots$$

Substituting their respective values,

$$\frac{P}{\gamma-1}\cdot\frac{dv}{J}=\frac{p_1}{\gamma_1-1}\cdot\frac{dv}{J}+\frac{p_2}{\gamma_2-1}\cdot\frac{dv}{J}+\ldots\ldots$$

$$\text{or}\quad \frac{P}{\gamma-1}=\frac{p_1}{\gamma_1-1}+\frac{p_2}{\gamma_2-1}+\ldots\ldots$$

are twofold—(1) Observations made in open air and (2) Observations made on air contained in tubes.

(1) **Open air observation**—(*a*) The earliest experimental determinations of the velocity of sound were made by Mersenne (1636) and by Gassendi (1658). A gun was fired at a particular place and at a distant place observations were made of the instant of time when the flash of light was seen and also of the instant of time when the report of the gun was heard. As the propagation of light is almost instantaneous, the interval of time between these two instants recorded by a watch was evidently taken by the sound to travel the distance between the two places. Measuring the distance between the two places and dividing the distance by the interval of time noted in the watch, the velocity of sound was calculated. The method is sometimes called the ***signal method***. The velocity thus determined was subject to two errors :—(1) *wind effect.* observed in the Article 46 (*d*), and (2) the error, depending on the quickness of perception and response in recording the two instants of time, which is known as '***human effect***' or *the personal equation* of the observer.

(*b*) To eliminate the effect of wind, certain members of the Paris Academy made a fresh determination of the velocity of sound in 1738 by what is known as the method of '*reciprocal observation.*' Cannons were fired alternately from two stations at a distance of 18 miles at intervals of half an hour. The interval of time between the flash and the report was noted at each station by the observers. The mean of any two intervals of time recorded at the two stations is evidently the time taken by the sound to travel the distance between the two stations. The mean value for

the velocity calculated over a large number of observations was reduced to dry air at $0°C$. The value obtained was 332·2 metres per second. It will be observed that the determination is not free from the human error '*personal equation*.'

(c) The method of reciprocal observation was improved by Regnault in 1864 to get rid of the personal equation by electrical registration apparatus.

In his apparatus shown in fig. 11, a drum D is rotated and translated at a constant speed by a clock-work. On the

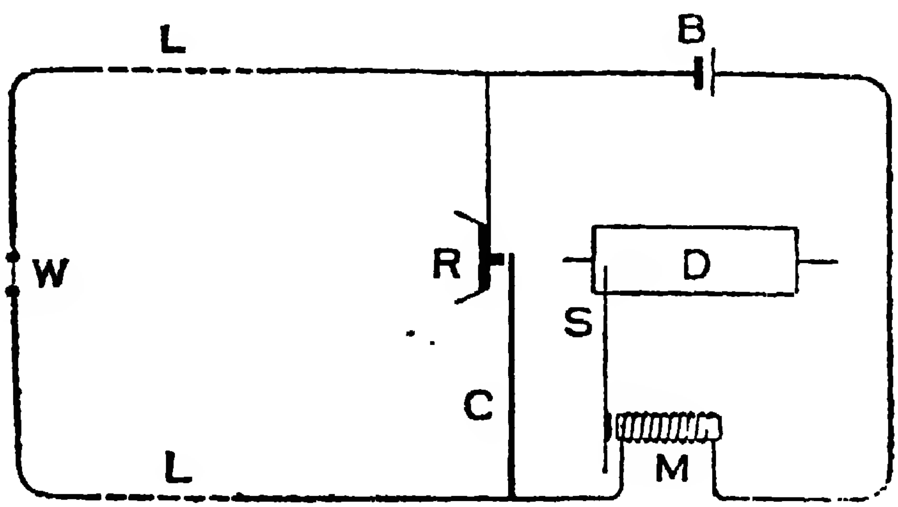


FIG. 11

drum a style S makes a mark when attracted by the electromagnet M. Current from a battery B traverses the line L extending between two distant stations and excites the electromagnet which normally attracts the style S.

At the sending station a gun is fired; a wire W, of the line, stretched before the muzzle of the gun is broken by the firing of the gun. This opens the circuit and the style being no longer attracted by electromagnet, ceases to make its usual mark on the drum. The arrival of the sound wave at the receiving station pushes a receiving membrane R

connected to one end of the battery against a conductor C connected to the other end of the battery through the electromagnet. This re-establishes the current momentarily. The style being attracted by the electromagnet makes a mark on the drum. The interval between the times when the style discontinued its usual mark on the drum and when the momentary mark was made, is evidently the time taken by sound to travel the distance between the two stations. The time is measured from the speed of rotation of the drum. Repeating similar observations at the two stations the effect of wind was eliminated out and the velocity of sound calculated.

Regnault found that his apparatus also had a personal equation which he calculated and allowed for.

Bosscha (1858) and Hebb (1905) also determined the velocity of sound eliminating personal equation.

(*d*) *Recent Determination*—A determination of the velocity of sound has been made by Esclangon in 1918. The method is analogous to that of Regnault. The sound was produced by firing a gun and the waves were received by a hot wire microphone. The arrival of the sound wave was recorded on an Einthoven string oscillograph.

Angerer and Ladenburg have also made a determination of the velocity of sound by a similar method in 1922. The sound was produced by firing a charge of powder, which broke a wire of the recording circuit. The sound waves were received by a microphone and recorded by an Einthoven string oscillograph.

The accepted value for the velocity of sound is 337·16 metres per second at $10°C$. in dry air.

* See 'A Text-Book of Sound,' Wood, p. 444.

(2) Velocity of Sound in Gases contained in Tubes—The velocity of sound in air and other gases has been found out with the air or gas enclosed in tubes, as in the *air resonator* due to Wertheim (Art. 95) and in *Kundt's tube* (Art. 97). The advantage of this method over the open air observation lies in the fact that the experiment is carried out in the laboratory with a small quantity of gas, under controlled conditions of temperature and other factors influencing the velocity of sound. The method is based upon resonance and will be treated in detail in Chapter XI on resonance and response.

49. Amplitude and Velocity of Propagation—Regnault showed by his experiments that the speed of sound increased with the increase of amplitude or intensity of the sound. By the method described above he determined the velocity of sound emitted by the same source over two distances 1280 and 2445 metres respectively. His results were that when the distance between the gun and the receiving station was 1280 metres, the velocity was 331·37 metres per second and when the distance was 2445 metres, the velocity was 330·7 metres per second.* Hence it is evident that when the gun was nearer the receiving station and the intensity of the sound consequently greater, the velocity of sound is larger than when the gun was at a greater distance from the receiving station and the intensity of the sound consequently less. This is because of the fact that when the intensity of sound is larger, the temperature in the compressions is higher and the velocity of propagation is accordingly increased.

The increase in the velocity of propagation with the

*Data taken from the 'Dictionary of Applied Physics'

increase in intensity of sound is beautifully demonstrated by the photographs of bullets in flight taken by Professor C. V. Boys in 1892.*

50. Frequency and the Velocity of Propagation—

The velocity of sound in a medium, as has been found in Art. 43, depends on the properties (*viz.* elasticity and density) of the medium only. Hence whatever the frequency of the source may be, the wave-length of the waves emitted by the source adjusts itself to have one and the same value for the velocity of propagation of the waves in the same medium (*cf.* $n\lambda = v$).

Pierce, however, recently observes the velocity of sound in free air at $0^\circ C$ for frequencies 1,000 and 50,000 amounts to 332·94 and 332·74 metres per second nearly, and the velocity in carbon dioxide at $0^\circ C$ for frequencies 42,000 and 200,000 amounts to 258·52 and 260·15 metres respectively. The theoretical explanation of these results seems not to be clear, they appear to suggest that the adiabatic condition, during the propagation of sound waves as supposed by Laplace, does not strictly hold good for low frequencies of the source.

Examples

1. Describe any two methods of determining the velocity of sound in air. Explain the various steps of the process in each case.

How does the velocity of sound depend on the temperature of the medium? (*C. U. 1913*)

2. Find the velocity of sound in air (at any moment), from the following data :—

Pressure of the atmosphere = that of 760 millimetres of mercury.

Density of air = 0·001208.

Ratio of the specific heats = 1·41.

Temperature of the air = $10^\circ C$.

*See 'A Text-book of Sound," Wood, 270.

$\therefore\ 2{\cdot}8 = \frac{10^5}{33\times10^3} - \frac{10^5}{V}$ where V = velocity in iron

A sound is emitted by a source placed at one end of a long iron tube and two sounds are heard at the other end at an interval of 2·5 seconds. If the length of the tube is 951·26 metres, find the velocity of sound in iron. (*C. U. 1914*)* 1914 ✓

3. A litre of hydrogen at normal temperature and pressure weighs 0·0896 gramme. Find the velocity of sound in hydrogen at a temperature of 16 C, when the pressure is 750 mm., the ratio of the specific heats (of hydrogen) being 1·4. [Density of mercury = 13·6 and g = 980]. (*C. U. 1915*)

4. State how Newton tried to explain the discrepancy between the velocity of sound as determined by experiment and as determined from his expression $V = \sqrt{P/\rho}$.

What was Laplace's correction?

Determine the velocity of sound at N. T. P., and deduce the change in velocity per centigrade degree rise in temperature if $a = 0{\cdot}00367$. (*C. U. 1917*)

5. Obtain an expression for the velocity of sound in air, in terms of the pressure and the density.

Calculate the velocity of sound in air at 30° C., when the barometric height (corrected) is 775 mm.

$a = 0{\cdot}00367$ $\gamma = 1{\cdot}4$ (*C. U. 1918*)

6. Mention the different factors or sources of error that influence the velocity of sound as determined by open-air observations and explain how they may be corrected in order to obtain an absolute value. (*C. U. 1921*)

7. Find an expression for the velocity of sound in air from the theory of dimensions, assuming that it depends only on pressure and density.† Is the formula so obtained verified experimentally? Give reasons for your answer.

Calculate the velocity of sound in air at 0°C and pressure of 760

* See Art. 60.

† See 'Text-Book of General Physics' by the author, Art. 147(2).

millimetres of mercury, given that the corresponding density of air is 0·001298. $g = 981$, and specific gravity of mercury is 13·6. (*C. U. 1924*)

8. Give Newton's formula for the velocity of sound in air. In what respect was it defective, and what correction was it found necessary to make that it might give accurate results? (*C. U. 1925*)

9. State the law connecting the velocity of sound through a gas with its temperature and pressure.

If the velocity of sound through hydrogen at 0°C is 4,200 feet per second, what will be the velocity of sound (at the same temperature) through a mixture of two parts of volume of hydrogen to one of oxygen? The density of oxygen is sixteen times that of hydrogen). (*C. U. 1921*)

10. Show that the velocity of propagation of sound waves in a gas is given by the expression $\sqrt{\dfrac{E}{\rho}}$, where E is the elasticity and ρ is the density of the gas. What expression will you use for value of E? Give your answers for choosing this particular value. (*C. U. 1930*)

11. Derive a general expression for the velocity of sound in a gas and also discuss formula due to Newton and Laplace.

Calculate the velocity of sound in air at N. T. P. (The ratio of specific heats for air = 1·4). (*C. U. 1933*)

12. Give a general expression for the velocity of sound in a gas and discuss the formula due to Newton and Laplace.

Calculate the velocity of sound in air at N. T. P. (ratio of specific heats = 1·41). How does the velocity vary with pressure? Give reasons for your answer. (*C. U. 1947*)

13. Give an account of the classic determination of the velocity of sound in free air and state the results obtained (*C. U. 1949*)

14. Derive an expression for the velocity of sound in gases. Describe any method of determining the velocity of sound in air.

(*C. U. 1950*)

CHAPTER VII

DOPPLER'S PRINCIPLE

51. When there takes place a relative motion between a source of sound and an observer, the pitch of the sound emitted by the source appears to change. This was first explained by Doppler, who first applied it to the change of colour of certain stars as they moved in the line of sight.

With sound the effect is commonly observed by a person who stands at a railway station and a locomotive, while whistling, approaches or passes away from him. As the engine approaches the observer, it follows up the waves sent out by it towards the observer previously and hence crowds a larger number of waves in a given length than if it were at rest. The number of waves received by the observer in a certain time is thus greater than the number of waves sent out by the source in the same time. Hence the *pitch* of the sound appears to rise. A similar effect is produced if the source remains stationary but the observer moves towards the source or if both the source and the observer move towards each other.

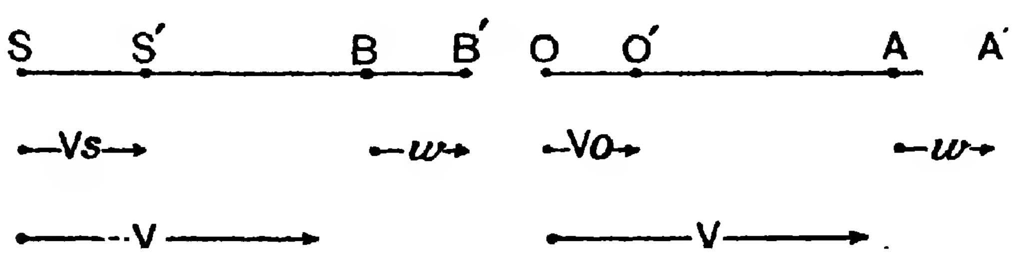
On the other hand, if the locomotive passes away from a stationary observer, it draws away from the waves sent out by it towards the observer previously and hence sends a smaller number of waves in a given length than if it were at rest. The number of waves received by the observer in a given time is thus less than the number of waves sent out by the source in the same time. Hence the *pitch* of the sound appears to fall. A similar effect is produced if the

source remains stationary but the observer moves away from the source or if the source and the observer move away from each other.

If both the source and the observer remain stationary but the wind blows from the source torwards the observer, a crowding of waves will take place towards the observer giving rise to an apparent increase in the pitch of the source. If, on the other hand, the wind blows from the observer to the source, there will take place an apparent fall in the pitch of the source.

52. Source, Observer and Medium in Motion—The change in pitch of the source due to a motion of the source, the observer and the medium can be calculated as follows.*

In fig. 12, S and O respectively represent the positions of the source and the observer at a particular instant of time. In making the calculation, we shall suppose that the source, the observer and the medium all are moving from left to right in the positive direction of x, so that the source moves towards the observer, the observer moves away from



the source and the wind blows from the source to the observer. If, in any particular case, the source moves away from the observer, or the observer moves towards

*Rayleigh has pointed out the difficulty of the calculation due to a moving source. 'Theroy of Sound,' Vol. 11. p. 159.

the source or the wind blows from the observer towards the source, their velocities will have to be taken with a negative sign.

In figure 12, SS' represents the distance passed over by the source in one second or the velocity V_S of the source, $OO' = V_O$, the velocity of the observer, $BB' = AA' = w$, the velocity of wind ; and $SB = OA = V$, the velocity of sound waves in still air.

Looking to observer.

(1) Let a wave reach the observer at a particular instant of time when the observer is at O. After the lapse of one second from this instant of time, the wave will reach A' ; for the wave travels a distance OA in one second in still air and the air moves a distance AA' in one second. Of all the waves received by the observer in *this* second, the last one is received when the observer is at O'; for in one second the observer moves from O to O'. Thus all the waves received by the observer in one second are contained within the length $O'A'$ or $V+w-V_O$.

(2) Looking to the source, the wave which the source emits at a particular instant of time when the source is at S will be at B' after the lapse of one second from this instant of time ; for the wave travels a distance SB in one second in still air and the air travels a distance BB' in one second. Of all the waves sent out by the source in *this* second, the last one is sent out when the source is at S' ; for in one second the source travels from S to S'. Thus all the waves sent out by the source in one second are contained within the length $S'B'$ or $V+w-V_S$.

If n be the real frequency of the source, it sends out n waves in one second which are contained within the

length $V+w-V_s$. If n' be the apparent frequency as heard by the observer, n'* is the number of waves contained within the length $O'A'$ or $V+w-V_0$ and we evidently have,

$$\frac{V+w-V_s}{n} = \frac{V+w-V_0}{n'} \qquad n'=n\frac{V+w-V_0}{V+w-V_s}$$

53. Source in Motion, Observer Stationary—Let the source while emitting sound move towards a stationary observer. Let also V be the velocity of sound, V_s that of the source, n its real pitch and n' the apparent pitch as heard by the observer.

The effect produced is a change of wave-length of the sound waves as heard by the observer. For in absence of motion of the source the n waves emitted by it in one second would have been contained within a length V, but due to the motion of the source towards the observer, a crowding of the waves takes place towards the observer, and hence the wave-length of the sound waves is decreased towards the direction of the observer. To calculate the wave-length λ' in the direction of the observer, we see that the first wave emitted by the source while at S (fig. 12) at a particular instant of time travels to B a distance V in one second from this instant of time. Again the last wave of this second is emitted by the source while at S' for the source in one second moves a distance SS' and emits the last wave of the second at S'. Hence the n waves are contained in the length $S'B$ or $V-V_s$

The wave-length $\lambda'=\dfrac{V-V_s}{n}$

The apparent pitch $n'=\dfrac{V}{\lambda'}=\dfrac{V}{(V-V_s)/n}=n\dfrac{V}{V-V_s}$

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*Art. 20

If the wind blows from the source to the observer, the effect is to increase V by w where w is the velocity of the wind.

Hence the apparent pitch $n' = n \dfrac{V+w}{V+w-V_s}$

54. Source Stationary, Observer in Motion.—In this case as the source remains stationary, the wave-length of the sound waves is not altered, but if the observer moves away from the source, he misses some of the waves and thus in a second receives a fewer number of waves than what are sent by the sources.

In fig. 12 let S be the stationary source and O the position of the observer at a particular time. The observer is moving away from the source with a velocity V_0. The wave which the observer hears at O moves to A a distance V in one second. Also due to the motion of the observer from O to O' in one second, he receives the waves contained in the length $O'A$ in one second and misses the waves within the length OO'. Since n is the frequency of the source, n waves are contained in the length V. Hence the number of waves in the length $O'A$ or the apparent pitch

$$n' = \frac{n}{V} . O'A = n \frac{V-V_0}{V}$$

If the wind blows from the source to the observer, the value of V is to be increased by the velocity of the wind w.

Hence apparent pitch $n' = n \dfrac{V+w-V_0}{V+w}$

55. Source and Observer in Motion.—In this case both the effects produced by the moving source and moving observer are present.

If V be the velocity of sound, V_s that of the source moving towards the observer and n the frequency of sound emitted by the source, the n waves will be contained within a length $V-V_s$ (art. 53). Hence the wave-length of the sound is changed to λ' where

$$\lambda' = \frac{V-V_s}{n} \quad \ldots \ldots \ldots \ldots \ldots \quad (\mathrm{i})$$

Also if the observer moves away from the source with a velocity V_o, he in one second will receive the waves contained within a length $V-V_o$ of the medium (art 54). Hence the pitch n' of the sound heard by the observer will be given by

$$n' = \frac{V-V_o}{\lambda'} = \frac{V-V_o}{V-V_s} = n\frac{V-V_o}{V-V_s} \quad \ldots \ldots \text{ from (i)}$$

If the wind blows from the source to the observer, the effect is to increase V by w the velocity of wind. Hence,

$$n' = n\frac{V+w-V_o}{V+w-V_s}$$

The deduction of arts. 53, 54 and 55 include that the source is moving towards the observer, the observer away from the source and the wind from the source towards the observer along the line of motion of the source or observer. The sign of one or more of these velocities will be considered negative if the direction of it is opposite to what has been assumed in course of the treatment.

56. Examples.—As an illustration to the application of the above formula, the following numerical examples may be considered.

(1) *A locomotive passing an observer at 80 miles an hour is sounding its whistle. The velocity of sound being 1120 feet per second, determine the interval (ratio of the frequencies) between the notes heard as the engine approaches and recedes.*

Here the observer and the medium are supposed not to be in motion, therefore in the equation of the previous article $V_0=0$ and $w=0$. Let n_1 be the apparent pitch when the engine approaches the observer, n_2 that when the engine recedes from the observer and n the real frequency of the source.

(The velocity 80 miles per hour is equivalent to 117·33 feet per second.)

$$= n\frac{V}{V-V_s} = n\frac{1120}{1120-117{\cdot}33} = n\frac{1120}{1002{\cdot}67} \qquad (1)$$

When the source recedes away, V_s in the above formula must be taken with a negative sign as the formula is deduced on the assumption that the source moves towards the observer.

$$\therefore \quad n_2 = n\frac{V}{V+V_s} = n\frac{1120}{1120+117{\cdot}33} = n\frac{1120}{1237{\cdot}33} \ldots (2)$$

Hence the interval between the two notes.

$$\frac{n_1}{n_2} = \frac{1237{\cdot}33}{1002{\cdot}67} = 1{\cdot}23 \text{ nearly.}$$

(2) *An engine in a cutting between two bridges, is whistling when its velocity is $\frac{1}{25}$th of sound in air. Compare the frequencies of the echoes from the two bridges to an observer between them.*

(*B. Sc. London University, 1896*)

Let A and B represent the two bridges and that the engine is moving in the cutting from A to B. Let n be the

frequency of the whistle, n_1 and n_2 the apparent frequencies of the waves reflected from A and B respectively.

$$n_1 = n\frac{V}{V+V_s}$$ (as the source is moving away from A the velocity of source V_s will be taken with a negative sign)

$$= n\frac{V}{V+\frac{V}{20}} = n\frac{20}{21}$$

$$n_2 = n\frac{V}{V-V_s}$$ (as the engine is moving towards B, the velocity V_s will be taken with a positive sign)

$$= n\frac{V}{V-\frac{V}{20}} = n\frac{20}{19}$$

$$\therefore \quad \frac{n_2}{n_1} = \frac{21}{19} = 1\cdot1053 \text{ approximately.}$$

57. Experiments of Doppler's Principle—Doppler's principle has been verified in a number of ways. In the earliest experiments of Buijs Ballot and Scott Russel (1845), the alteration of pitch of musical instruments carried on locomotives was examined.

A laboratory method of observing the effects was devised by Mach (1861) in which a long tube (6 ft. long) was provided with a whistle at one of its ends. The tube was rotated about an axis through its centre and the whistle was blown by wind forced along the axis of the tube. An observer, situated in the plane of rotation of the tube, hears the fluctuation in pitch of the sound emitted by the whistle as the whistle approached and receded from him.

Konig (1865) mounted two tuning forks on resonant cases. If set to vibrations these tuning forks produced four beats per second when they were stationary. One of the sounding tuning forks was then made to approach or recede from the observer while the other was kept stationary. Due to the apparent change in frequency of the moving fork the frequency of beats was changed. Noting the change in frequency Doppler's principle was verified.

Doppler's principle has been recently demonstrated by S. R. Humby in which two telephones were actuated by an oscillating valve circuit. One of the telephones was then moved. The motion produced a change of frequency and the beats formed between the two were observed by a sensitive flame.

Doppler's principle has found an important application in finding the velocity of a star in the line of sight and in many astronomical investigations The spectrum of a star generally consists of bright and dark lines. If a star approaches the earth, the apparent pitch of the light emitted by it increases and the spectral lines are displaced towards the violet end of the spectrum. If, on the other hand, the star recedes from the earth, its apparent pitch falls and the spectral lines are displaced towards the red end. From a measurement of the displacement, the velocity of the star can be found out.

Examples

1. Explain Doppler's principle, and describe how it can be easily demonstrated.

Two trains are approaching from opposite directions with the same speed of 100 ft. per sec. The whistle of the first train is of frequency 1024. Find the variation of the apparent pitch calculated by an observer in the second train as the trains pass, supposing there is no wind and that the velocity of sound in air is 1100 ft. per sec. *(C. U. 1918)*

2. Explain Doppler's principle and how it can be demonstrated.

Show that, in the absence of wind, a given speed of approach of the source raises the apparent pitch more than the same speed of approach of the recipient. *(C. U. 1920)*

3. Explain Doppler's principle.

A man standing by a railway notices the change of pitch of the note due to the whistle of an engine as it passes by him. If the

$1093(1+\cdot 0036\times 24)$

$2000\times\frac{1093(1+\cdot 0036\times 24)}{1093(1+\cdot 0036\times 24)-88}$

frequency of the whistle be 256 vibrations per second and the velocity of the engine be $\frac{1}{20}$th of that of sound, what will be the frequencies of the notes heard by the man before and after the engine passes him ? *(C. U. 1926)*

4. Explain how the pitch of the sound alters with the motion of the source or the observer.

Deduce a formula connecting the pitch of the note heard with the velocities of the source and the observer. *(C. U. 1927)*

5. *The pitch of a musical note is altered if the source or the observer move relatively to one another.* Explain clearly the reason of this phenomenon. Deduce an expression showing the relation between the alteration of pitch and the velocities of the source, the observer and the medium. *(C. U. 1929)*

6. What is Doppler's principle in sound ? Show that if the source moves away with the velocity of sound from an observer who is at rest, the frequency of the vibration heard is halved. *(C. U. 1937)*

7. Explain Doppler's principle and describe how it can be demonstrated. A locomotive whistle emitting 2000 waves per sec. is moving towards you at the rate of 60 miles an hour on a day when the thermometer stands at 24°C. Calculate the apparent pitch of the whistle (Vel. of sound in air at 0°C = 1093 ft./sec.) *(C. U. 1940)*

8. What is Doppler's effect ? Derive and explain the formula for the apparent frequency due to it.

A train is passing a railway station with a speed of 40 m. p. h. and blowing continuously a whistle of frequency 256 per sec. What will be the frequencies apparent to a person waiting at the platform ? When the train is (a) approaching, (b) departing ? What is the interval between these two notes ? (Vel. of sound = 1120 ft. per sec.) *(C. U. 1945)*

9. What is Doppler's principle ? Show how the observed frequency of waves will differ from the frequency at which they are emitted if the source is approaching a moving observer.

A spectroscopic examination of light from a certain star shows that the apparent wave-length of a certain spectral line is 5001 A. U., whereas the observed wave-length of the same line produced by terrestrial source is 5000 A. U. In what direction and at what speed do these figures suggest that the star is moving relative to the earth ? *(C.U. 1947)*

10. How does the motion of the source of sound affect the apparent pitch of the note heard ? *(C. U. 1950)*

11. What is Doppler's principle in sound ? Show that if the source moves away from an observer who is at *rest*, the frequency of vibrations heard is halved. *(C. U. 1952)*

12. Explain clearly Doppler effect in sound. Show that the Doppler effect is greater when the source approaches the observer than when the observer approaches the source with the same speed. see P. 2

Calculate the velocity at which a source of frequency 10 thousand per second should approach the observer at rest in order to produce a Doppler shift of 200 per second.

CHAPTER VIII

VELOCITY OF LONGITUDINAL WAVES THROUGH SOLIDS AND LIQUIDS

58. Velocity of Longitudinal Waves through Solids—In obtaining a theoretical expression for the velocity of propagation of longitudinal waves through a solid medium, we shall follow the same procedure as in Art. 43 for finding the velocity of propagation of longitudinal waves in a gas.

Consider a rod of some isotropic solid material having unit sectional area. Let the longitudinal waves travel through it in the direction AB (fig. 9) with a velocity V. For convenience consider two plane sections of the rod at A and B fixed in space, such that the former is a region of normal pressure and the latter is a region of rarefaction. Let the rod be moved backwards in the direction BA with a velocity V, such that the longitudinal waves in the region AB (in space) cease to advance. Hence the velocity of the particles of the medium, their states of displacement, the pressure and the density of the medium, though different at different sections of the region AB, retain their values throughout the time. Since the plane section at A is a region of normal pressure, the particles of the solid will have no velocity of their own but due to the backward velocity of the rod, a volume V of the medium will pass out of the region AB per second through the section at A. On the other hand, the section at B being a region of rarefaction the particles of the solid will have a velocity of their own from right to left. This combined with the velocity of the

rod, a volume V', greater than V, will enter into the region AB per second through the plane section at B.

But because the density of the medium at any point within the region AB remains unaltered, the mass of the solid in the region remains the same, or the mass of the solid leaving the region per second through A is the same at the mass of the solid entering the region per second through B. Hence if ρ and ρ' represent the densities of the medium at A and B respectively, we have the so-called equation of continuity,

$$V\rho = V'\rho' \qquad \dots \qquad \dots \qquad \dots \qquad (1)$$

Again, although the masses of the solid crossing AD and BC per second are the same, still they possess different momenta due to their unequal velocities. The momentum gained by the region AB per second through B is $V'\rho'.V'$ or $V'^2\rho'$ while that lost through A per second is $V\rho.V$ or $V^2\rho$. Hence the net gain of momentum per second by the region $AB = V'^2\rho' - V^2\rho$.

By Newton's second law of motion, this must be equal to the external force on the region AB. Since the sectional area of the rod is unity, this external force is the difference of pressures between the plane sections at A and B. Hence if P and P' be the pressures at A and B respectively,

$$P - P' = V'^2\rho' - V^2\rho \qquad \dots \qquad \dots \qquad (2)$$

Substituting the value of V' deduced from (1) in (2)

$$P - P' = \frac{V^2\rho^2}{\rho'} - V^2\rho$$

$$= V^2\rho\left(\frac{\rho}{\rho'} - 1\right) = V^2\rho\frac{\rho - \rho'}{\rho'}$$

$$V^2 = \frac{1}{\rho} \cdot \frac{P - P'}{\dfrac{\rho - \rho'}{\rho'}}$$

Let l and l' represent the lengths per unit mass at A and B respectively, the sectional area being supposed unaltered.

The specific volume at $A = l \times 1 = \frac{1}{\rho}$, or $\rho = \frac{1}{l}$

and the specific volume at $B = l' \times 1 = \frac{1}{\rho'}$, or $\rho' = \frac{1}{l'}$

$$\therefore \quad V^2 = \frac{1}{\rho} \cdot \frac{P - P'}{\dfrac{\frac{1}{l} - \frac{1}{l'}}{\frac{1}{l'}}} = \frac{1}{\rho} \cdot \frac{P - P'}{\dfrac{l' - l}{l}}$$

But $\dfrac{P - P'}{\dfrac{l' - l}{l}}$ measures the ratio between longitudinal stress and longitudinal strain which is the Young's modulus Y of the solid.

Hence $V^2 = \dfrac{Y}{\rho}$

or $V = \sqrt{\dfrac{Y}{\rho}}$

59. *Alternative Method*—An alternative treatment based on the same lines as Art. 44 can be given for the velocity of propagation of longitudinal waves through a solid.

Consider a uniform rod of unit sectional area of the solid medium and consider two plane sections A and B of the rod (fig. 10).

Let the positions of A and B be defined by their distances from some fixed point to the left of A. Let these distances be represented by x and $x + \delta x$, so that the positions of A and B are respectively defined by the co-ordinates x and $x + \delta x$ respectively. The length of the slice AB is thus δx.

Let the arrival of a longitudinal wave propagating in the direction of x subsequently displace the plane section A' through y. Since in a longitudinal wave, the displacements of the particles of the medium and the direction of propagation of the wave are along the same line, the actual position of the plane section measured from the same fixed point will be given by the co-ordinate $x+y$. This is shown by A' in figure 10. Again, as y measures the displacement in the direction of x, $\frac{dy}{dx}$ measures the rate of displacement with respect to distance along direction of x. Hence the change of displacement between the plane sections A and $B = \frac{dy}{dx}\delta x$

and the displacement of the plane section $B = y + \frac{dy}{dx}\delta x$

The actual position of the plane section B is thus

$$= x + \delta x + y + \frac{dy}{dx}\delta x$$

This is shown by B' in fig. 10.

The length of the slice in the displaced position

$$= \delta x + \frac{dy}{dx}\delta x$$

$\therefore$ the change in length of the slice

$$= \delta x + \frac{dy}{dx}\delta x - \delta x = \frac{dy}{dx}\delta x$$

and the fractional change in length or the longitudinal strain

$$= \frac{\frac{dy}{dx}\delta x}{\delta x} = \frac{dy}{dx}$$

If F represents the stretching force over the plane

section A (of unit area), Young's modulus Y of the solid is given by

$$Y = \frac{F}{\frac{dy}{dx}}$$

$$\text{or} \quad F = Y\frac{dy}{dx} \quad \dots \quad \dots \quad \dots \quad (1)$$

The stretching force over the plane section B

$$= F + \frac{dF}{dx}\delta x = Y\frac{dy}{dx} + \frac{d}{dx}\left(Y\frac{dy}{dx}\right)\delta x$$

$$= Y\frac{dy}{dx} + Y\frac{d^2y}{dx^2}\delta x \quad \dots \quad \dots \quad \dots \quad (2)$$

(as Y does not change with x)

From (1) and (2), the difference between the stretching forces at A and B

$$= Y\frac{d^2y}{dx^2}\delta x$$

This is the moving force on the slice AB of the solid. The mass of the moving slice $= 1 \times \delta x \times \rho$, where ρ is the density of the solid.

$\therefore$ by Newton's second law of motion,

$$Y\frac{d^2y}{dx^2}\delta x = 1 \times \delta x \times \rho\frac{d^2y}{dt^2}$$

as $\frac{d^2y}{dt^2}$ measures the acceleration of motion.

$$\therefore \quad \frac{d^2y}{dt^2} = \frac{Y}{\rho}\cdot\frac{d^2y}{dx^2}$$

This equation (D' Alembert's equation) is similar to that obtained in Art. 44. The solution of this is similar, which shows that the velocity of propagation is given by

$$V = \sqrt{\frac{Y}{\rho}}$$

60. Experiments on the Velocity of Longitudinal Waves in Solids—(*a*) The earliest determination of the velocity of propagation of longitudinal waves in solids was made by Biot. He took a long cast iron pipe to one end of which a bell was mounted. The bell was struck by a hammer. Placing the ear near the other end of the pipe, two sounds were heard. As sound travels with a much larger velocity through solids than through gases, the first sound was evidently that propagated through iron and the second propagated through the air in the pipe. He noted the interval of time between the two sounds heard. If l represents the length of the pipe and V_a and V_i the velocities of longitudinal waves through air and iron respectively, the time taken by the longitudinal waves to travel through the air in the pipe is $\frac{l}{V_a}$ and that through the material of the pipe is $\frac{l}{V_i}$. The interval of time between the two sounds is thus $\frac{l}{V_a}-\frac{l}{V_i}$. Hence if t be the interval, we have

$$t=\frac{l}{V_a}-\frac{l}{V_i}=l\left(\frac{1}{V_a}-\frac{1}{V_i}\right)$$

Biot measured t and l, and from a knowledge of V_a calculated V_i, the velocity of longitudinal waves through iron. He used 376 pipes of cast-iron forming a total length of about 951 metres. The velocity obtained in cast-iron was 3500 metres per second.

It will be observed that the determination of t involved the error due to 'personal equation' as in the case of the open air observation in finding the velocity of sound through air.

(*b*) The velocity of longitudinal waves in solids is more conveniently determined by a method devised by Kundt in 1866 and based upon the principle of resonance. The method is described in detail in Chapter XI.

61. Velocity of Longitudinal Waves in Liquids—The formula of Newton, *viz.* $V=\sqrt{\frac{E}{\rho}}$ for the propagation of longitudinal waves in a gaseous medium is equally applicable to liquids. Strictly speaking the modulus of adiabatic volume elasticity must be taken for E. Experimental determination for the modulus of bulk elasticity of a liquid are mostly static measurements and these therefore determine the value of E under isothermal conditions. Thermodynamical considerations show that for any substance the modulus of adiabatic volume elasticity is γ times the modulus of isothermal volume elasticity,* where γ is the ratio of the specific heats of the substance at constant pressure and at constant volume.† Hence from a determination of the isothermal volume elasticity of the liquid,‡ the adiabatic volume elasticity can be calculated and the theoretical value of the velocity of propagation of longitudinal waves in a liquid can be evaluated.

In the case of water, the value of γ is nearly unity so that the modulus of adiabatic elasticity is nearly the same

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*See 'Theory of Heat' Preston, page 716.

† For any substance $\gamma=\frac{1}{1-a^2 kvT/C_p}$. (See Poynting and Thomson's Heat, page 289.)

Where a = coefficient of cubical expansion of the substance.

k = modulus of isothermal volume elasticity.

v = specific volume. T = the absolute temperature,

and C_p = specific heat at constant pressure expressed in ergs.

‡ See 'A Text-Book of General Physics' by the author, Art. 105.

as the modulus of isothermal elasticity. The velocity of the longitudinal waves in water is thus given by the square root of the ratio between the modulus of isothermal elasticity and density.

62. Experiments on the Velocity of Longitudinal Wave in Water.—

(*a*) The earliest recorded observation on the velocity of propagation of longitudinal waves in water was made by Colladon and Strum in the lake of Geneva in 1827. Two boats were moored at a known distance apart. From one of the boats a bell was suspended under water. Sound was produced by striking the bell with a hammer operated by a lever from the boat. The hammer, as it struck the bell, ignited a quantity of a powder in the boat by the lever The flash of light was observed from the other boat and a stop watch started simultaneously. To receive the sound waves from the other boat, an ear trumpet which consists of a bent tube was suspended from it below water. The broad end of the ear trumpet which was under water, was closed with a thin plate and the narrow end was placed near the observer's ear. The stop watch was stopped when the sound was heard by the observer. The interval of time recorded by the stop watch was evidently the time taken by the longitudinal waves to travel in water through the distance between the boats. The velocity of sound in water was calculated by dividing the distance between the two boats by the interval of time recorded.

The value obtained by them at 8°C was 1435 metres per second which is in excellent agreement with the calculated value 1436·4 metres per second under the conditions of salinity of the lake and the depth at which the experiment was made.

(*b*) Among the recent determinations of the velocity of longitudinal waves in sea water, are those of Wood and Browne in the Dover channel (1920-22) and of Stephenson on the American coast in 1923. Their methods are almost alike and known as the *radio-acoustic* method. In Stephenson's experiment the sound was produced by detonating a half-kilogram bomb at a depth of 10 metres and at the time a radio signal, instead of a light signal as in Colladon and Sturm's experiment, was sent. The sound signals were received on five microphones, each placed at a distance of 10 miles from the source. The radio signal as well as the sound signal was registered by an Einthoven galvanometer on a photographic recorder. The experimental value 1453·5 metres per second under the conditions of the experiment differed from the calculated value by about 0·1 per cent.

(*c*) *Tube Method*—The tube methods of Wertheim and of Kundt for finding the velocity of sound in gases have been applied in the case of liquids. These methods will be considered in detail in Chapter XI.

Examples

1. Indicate some method of experimentally determining the velocity of sound in solids.

Young's modulus for steel is 210×10^{10}, and the specific gravity is 7·8. Find the velocity of propagation of sound through a steel bar. How would the temperature of the bar affect the result?

(*C. U. 1916*)

*The depth at which the experiment is made is an important factor in calculating the value of the velocity. For, with change of depth, the elasticity as well as the density of water will change.

2. Prove that $V=\sqrt{\frac{Y}{\rho}}$, where V is the velocity of propagation of longitudinal waves in a solid of Young's modulus Y and density ρ.

Explain briefly how by Kundt's apparatus the velocity can be experimentally determined. *(C. U. 1919)*

3. Find the expression for velocity of propagation of longitudinal vibrations in solid rod.

Describe some method of finding the coefficient of longitudinal elasticity by an acoustical experiment. *(C. U. 1923)*

4. Explain why sound travels faster and is heard at greater distances in liquids and in solids than in gases *(C. U. 1951)*

5. Deduce a theoretical formula for the velocity of sound in gases.

A sound is emitted by a source placed at one end of an iron tube, one kilometre long, and two sounds are heard at the other end at an interval of 2·8 sec. If the velocity of sound in air in the condition of the experiment is 330 metres per sec., find that for iron. *(C. U. 1942)*

6. Find an expression for the velocity of propagation of longitudinal vibration in a solid.

Describe some method of finding the Young's modulus of elasticity by an acoustical experiment. *(C. U. 1944)*

7. Derive an expression for the velocity of propagation of longitudinal waves in a solid.

Describe an experimental arrangement by which the result can be utilised to measure Young's modulus of the material of a rod.

(C. U. 1954)

CHAPTER IX

TRANSVERSE VIBRATION OF STRINGS AND BARS

63. String—Of all vibrating bodies, the strings are the most prominent. The strings have been used as the source of musical sound from the earliest times and even now they form the essential parts of many of the musical instruments, such as the pianoforte, the violin, the *setar*, the *esraj* etc. Lord Rayleigh defines the string as follows :—'The string of acoustics is a perfectly uniform and flexible filament of solid matter stretched between two fixed points—in fact an ideal body, never actually realised in practice though closely approximated to by most of the strings employed in music.'* Thus an ideal string is *perfectly uniform*, having a constant mass per unit length, called the *linear density*, all over the string and is *perfectly flexible*, its 'stiffness' being supposed negligible, *i. e.* it shall require no force to bend it.

64. Velocity of Transverse Waves in a String—When a string is stretched under a tension and a certain portion of it is displaced laterally, transverse waves are set up in the string. These transverse waves travel along the string with a velocity depending upon the tension of the string and its linear density. To deduce an expression for the velocity of the transverse waves consider a string which is stretched with a tension T. The string is displaced in a direction perpendicular to its length so as to execute transverse vibrations. A short length BCD (fig. 13) of the

* **Vide 'The Theory of Sound', Lord Rayleigh, vol, I. p. 170**

string near to the summit of the displaced position will be bent into the arc of a circle.

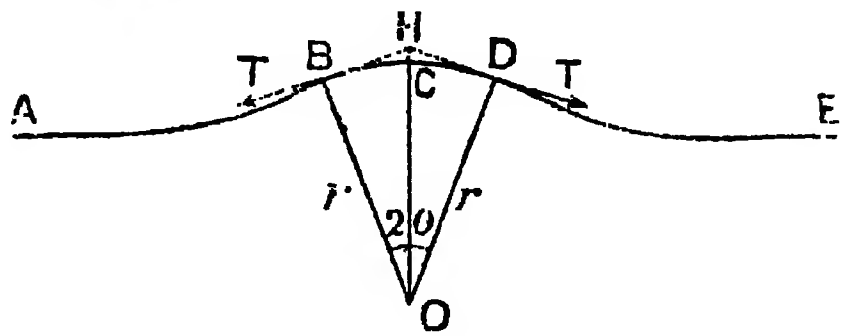

FIG. 13

The transverse wave propagating along the string from left to right with a velocity V (say) may be imagined by supposing the hump thus produced to travel with the same velocity. By moving the string with a velocity V from right to left the hump may be kept retained in space. For the circular motion of an element near the summit C of the hump, the necessary centripetal force is exerted by the tensions at the two ends B and D of the hump.

Let r be the radius of the circular arc into which BCD is bent and the radii and the two points B and D at equal distances from C meet at O. Also let the angle BOD be represented by 2θ. The straight line CO evidently bisects the angle BOD.

The two tensions, each of magnitude T, at B and D act along the tangents drawn to the circular arc at the respective points. The two tangents when produced backwards evidently meet at a point H, on the line OC produced. The components of the tensions at B and D, along CO are each equal to $T \sin \theta$ or $T\theta$ approximately, since θ is small. Again, if l be the length of the portion BCD of the string,

$$T\theta = T \frac{l}{2r}$$

The other components of the tensions perpendicular to CO cancel each other. Hence the resultant of the tensions at B and D is along CO and is of magnitude

$$2T\frac{l}{2r} = T\frac{l}{r} \quad \ldots \quad \ldots \quad (1)$$

If m represents the mass per unit length of the string the mass of the length BCD is ml and the centripetal force for its circular motion with a speed V is

$$ml\frac{V^2}{r} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Equating the two forces (1) and (2)

$$ml\frac{V^2}{r} = T\frac{l}{r}$$

whence $$V^2 = \frac{T}{m}$$

or $$V = \sqrt{\frac{T}{m}}$$

64. *Alternative Method.*—An alternative treatment involving the differential equation similar to Arts. 44 and 59 for the velocity of propagation of tranverse waves along a stretched string is noteworthy.

Let ABC (fig. 14) represent the undisplaced position of a string in the direction of x, fixed between the ends A and B and stretched with a tension T Let the string be subsequently displaced laterally in the direction of y so as to take up the position $APQB$. The displacement is supposed to be small so that the inclination of the string to the line AB is small.

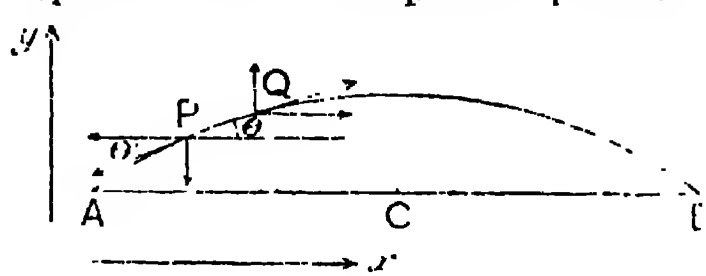

FIG. 14

Consider a short length δs of the string which may be supposed straight between two neighbouring points P and Q in the displaced position. The forces acting at P and Q are the tensions, each equal to T, along the tangents drawn to the string at P and Q as shown in the figure. If PQ is inclined at an angle θ to the direction of x, the tension T at P is equivalent to

$T\cos\theta$ along the negative direction of x

and $T\sin\theta$ along the negative direction of y.

Since θ is small, $\cos\theta$ may be approximately taken as unity and $\sin\theta=\tan\theta=\frac{dy}{dx}$. Therefore the above components of the tension at P are respectively equal to

T along the negative direction of x ... (1)

and $T\frac{dy}{dx}$ along the negative direction of y ... (2)

To calculate the two components of the tension T at Q we see from (1) above that the component along the direction of x is independent of x. Hence the component of the tension at Q along the direction of x is T but directed along the positive direction of x. The other component, which is along the positive direction of y, can be obtained from (2) above. Since $T\frac{dy}{dx}$ measure the y-component of the tension at P, the rate of change of the y-component of the tension along the x-axis $=\frac{d}{dx}\left(T\frac{dy}{dx}\right)$ and the change in the y-components of the tensions between P and Q

$$=\frac{d}{dx}\left(T\frac{dy}{dx}\right)\delta x$$

where δx is projection of PQ on the axis of x. Since θ is small, $\delta x=PQ=\delta s$.

$\therefore$ the change in the y-components of the tensions between P and Q

$$=\frac{d}{dx}\left(T\frac{dy}{dx}\right)\delta s$$

$$=T\frac{d^2y}{dx^2}\delta s \text{ (since } T \text{ is constant)}$$

$\therefore$ the y-component of the tension at Q

$$=T\frac{dy}{dx}+T\frac{d^2y}{dx^2}\delta s \quad \ldots \quad \ldots \quad (3)$$

To calculate the resulting force on PQ, we see that the forces along the x-direction at P and Q cancel each other while the forces along the y-direction at P and Q are equivalent to the difference of (2) and (3). Hence the resultant force on PQ

$$=T\frac{d^2y}{dx^2}\delta s^*$$

This provides the moving force on the length δs of the string. If m represents the mass per unit length of the string, $m.\delta s$ is the mass of the length δs and we have by Newton's second law of motion,

$$m.\delta s.\frac{d^2y}{dt^2}=T\frac{d^2y}{dx^2}\delta s$$

$$\left(\text{as } \frac{d^2y}{dt^2} \text{ measures the acceleration of the motion}\right)$$

or $\dfrac{d^2y}{dt^2}=\dfrac{T}{m}\cdot\dfrac{d^2y}{dx^2}$

The equation of motion is similar to that solved in

---

* It will be observed that this force acts along the negative direction of y; for over the displaced part APB of the string, $\frac{d^2y}{dx^2}$ is a negative quantity. Hence the resulting force $T\frac{d^2y}{dx^2}\delta s$ is directed along the negative direction of y.

Art. 44. The solution shows that the velocity of propagation of the transverse waves along the string is given by

$$V=\sqrt{\frac{T}{m}}.$$

66. Relative Velocities of Longitudinal and Transverse Waves in a String—It is found in Art. 58 or 59 that the velocity of longitudinal waves in a solid depends upon the Young's modulus and the density of the solid. Thus the velocity of longitudinal waves in a solid has a fixed value depending upon the material of the solid.

The velocity of transverse waves in a string depends upon the tension of the string. Thus by altering the tension of the string the velocity of transverse waves through it can be varied.

Let us now examine with what stretching force a string is to be stretched so that the transverse waves through it can travel, if possible, with the same velocity as the longitudinal waves through the string. Let a tension T be needed for the purpose.

The velocity of the transverse waves $=\sqrt{\frac{T}{m}}$

If ρ is the density of the material of the string and S its cross-section, the mass per unit length m is given by

$$m=S\times 1\times \rho$$

$\therefore$ velocity of the transverse waves $=\sqrt{\frac{T}{S\rho}}$ ... (1)

Again the velocity of longitudinal waves through it

$$=\sqrt{\frac{T}{\rho}}=\sqrt{\frac{YS}{\rho S}} \quad \dots \quad (2)$$

If the two velocities are equal, we have from (1) and (2)

$$T=YS \quad \dots \quad \dots \quad \dots \quad (3)$$

In other words, the tension needed to make the transverse waves through the wire travel, if possible, with a velocity equal to the velocity of longitudinal waves, is equal to the product of the Young's modulus of the string and its cross-section.

$$\text{From (3)} \quad Y = \frac{T}{S}$$

$\frac{T}{S}$ measures the stretching force per unit sectional area or the longitudinal stress. Again since

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}},$$

the longitudinal strain of the string under the tension T must be unity. In other words, the tension needed must be such that, if Hooke's law is held, the tension would stretch the wire to double its length, which is not possible.

Hence, no available tension can make the transverse waves travel through a stretched string as fast as the longitudinal waves through it.

67. Laws of Transverse Vibration of a Stretched String.—The result of Art. 64 or 65 for the velocity of propagation of transverse waves along a stretched string is directly applicable to the finding of the frequency of vibration of the string. If n represents the frequency of vibration and λ, the wave length of the transverse waves, the velocity of propagation V is given by

$$n\lambda = V$$

Substituting the value of V

$$n\lambda = \sqrt{\frac{T}{m}}$$

$$\text{or} \quad n = \frac{1}{\lambda}\sqrt{\frac{T}{m}} \qquad \ldots \qquad (1)$$

If a length l of the string is fixed between two points and the length vibrates in one segment having two nodes at the two ends and one antinode at the middle (fig. 15a). the length of the string

$$l = \frac{\lambda}{2} \text{ or } \lambda = 2l$$

and the relation (1) reduces to

$$n_1 = \frac{1}{2l}\sqrt{\frac{T}{m}} \quad \ldots \quad (2)$$

where n_1 is the frequency of vibration of the string.

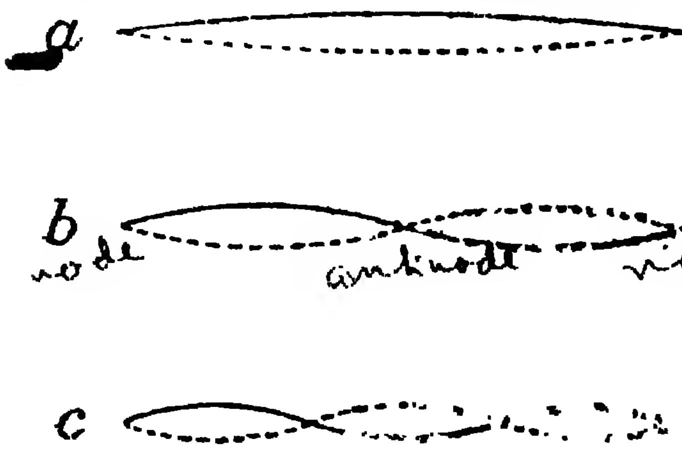

FIG. 15

If the string is made to vibrate in two segments* having two antinodes and three nodes (fig. 15, b), $l = \lambda$, and the relation (1) reduces to

$$n_2 = \frac{1}{l}\sqrt{\frac{T}{m}} \quad \ldots \quad \ldots \quad (3)$$

where n_2 is the frequency of vibration of the string.

If, on the other hand, the string is made to vibrate in three segments having three antinodes and four nodes (fig. 15, c) $l = \frac{3\lambda}{2}$ or $\lambda = \frac{2}{3}l$; and the relation (1) reduces to

$$n_3 = \frac{3}{2l}\sqrt{\frac{T}{m}} \quad \ldots \quad (4)$$

where n_3 is the frequency of the vibration of the string.

In general, if the string vibrates in s segments, there will be s antinodes separated by $(s+1)$ nodes and the

*See Art. 64.

length of the string $l = \frac{s\lambda}{2}$ or $\lambda = \frac{2}{s}l$. The frequency of vibration by relation (1) is given by

$$n_s = \frac{s}{2l}\sqrt{\frac{T}{m}} \qquad \dots \qquad \dots \qquad (5)$$

where n_s is the frequency of vibration of the string.

The laws of transverse vibration of strings embodied in equations (2), (3), (4) and (5) were discovered experimentally by Mersenne in 1638, hence they are sometimes called *Mersenne's laws*. They are three in number and can be stated as follows :—

(1) *Law of Length*

$$\text{Frequency} \propto \frac{1}{\text{length}}$$

if the tension and mass per unit length of the string are kept constant.

(2) *Law of Tension*

$$\text{Frequency} \propto \sqrt{\text{Tension}}$$

if the length and mass per unit length of the string are kept constant.

(3) *Law of Mass*

$$\text{Frequency} \propto \frac{1}{\sqrt{\text{mass per unit length of the string}}}$$

if the length and tension of the string are kept constant.

68. Note, Tone, Fundamental, Overtone and Harmonic—It has been found in Art. 67 that if a string stretched between two fixed points is set to vibrations, the frequency of vibration is lowest when the string vibrates in one segment. The string can be made to vibrate in one segment by displacing it laterally at its middle point. The string can be made to vibrate in two segments by displacing it laterally at a distance of *one-fourth* of its length

from one end and damping the vibrations at the *middle point* by lightly pressing at the middle point. The frequency of vibration in this case is doubled. In general, if the string is displaced at a distance of $\frac{1}{2s}$ th of its length from one end and pressed lightly at a distance of $\frac{1}{s}$ th of its length from the same end, the string vibrates in s segments and the frequency of vibration is s times that when the string is made to vibrate in one segment.

In fact, when a stringed musical instrument such as harp, mandolin, *setar*, pianoforte or violin is excited so as to produce a musical note, theoretically, the string vibrates having an infinite series of frequencies which are exact multiples of the lowest frequency. Some of these vibration frequencies are suppressed depending on the condition of exciting the string. If the string is excited by displacing it at its middle point, the middle point can no longer be a node. Hence, those tones of the infinite series which have their nodes at the middle point will not be produced. For example, if n be the frequency when the string vibrates in one segment having an antinode at the middle point thereby the tones of frequencies $2n$, $4n$, $6n$, etc., if produced, will have their nodes at the middle point. These tones cannot thus be produced when the string is struck at its middle point. Similarly, if the string is displaced at a distance of one-third of its length from *either* end, tones having frequencies $3n$, $6n$, $9n$, etc. will not be produced. In general, if a string is divided into s parts and displaced at any of these divisions, the tones having frequencies sn, $2sn$, $3sn$, etc. will not be produced. Thomas Young proved experi-

mentally* that when any point of a string is plucked, struck or bowed†, all the over-tones which require that point for a node will be absent from the resultant vibration.

A musical sound, as produced in the string, which comprises a number of frequencies is called a *note*, while a sound having a single frequency is called a *tone*. Thus, a *note* is a *complex* sound being a combination of tones and consisting of several frequencies. The note can be resolved into its constituent tones. On the other hand, a *tone* is a *simple* or *pure* sound having a single frequency and cannot be resolved. A tone is produced by a tunning-fork only—a fact, which makes it so important in acoustical experiments ‡.

In a complex sound, which is the combination of a number of tones, the tone having the lowest frequency is called the *fundamental* or *prime-tone* and the higher ones are called the *overtones* or *upper partial tones*.

We have seen that in the note emitted by a stringed musical instrument, the overtones are exact multiples of the fundamental in frequency. This is not usually the case with all musical instruments. In the note produced by the vibrations of a bell, the higher ones are not always the exact multiples of the fundamental in frequency. The term overtone or upper partial tone is used to denote any higher tone whether or not it is an exact multiple of the fundamental in frequency. A special term *harmonic-overtone* or *harmonic* is used to denote any of the overtones whose frequency is an

*See Tyndall's Sound. pages 118-124.

† These are the three principal modes of exciting a string, see Art. 145.

‡ See Art. 76.

an exact multiple of the frequency of the fundamental. Thus the more general overtone includes the harmonics but any overtone is not *necessarily* a harmonic. The overtones which are not harmonics of the fundamental are called *inharmonic overtones.* Out of the harmonics, that particular one of which the frequency is *double* the frequency of the fundamental is called the *first harmonic* or *octave* of the fundamental. The harmonic having a frequency equal to *three times* the frequency of the fundamental is called the *second harmonic* of the fundamental and so on for other harmonics. In general, if a harmonic has a frequency which is *s times* the frequency of the fundamental, it is called the $(s-1)$th *harmonic* of the fundamental.

69. Verification of the Laws of Transverse Vibration of a Stretched String—The laws of transverse vibration of a stretched string can be verified by (1) **Thompson's monochord** also called the **sonometer** and (2) **Melde's experiment.**

70. Thompson's Monochord or Sonometer—The

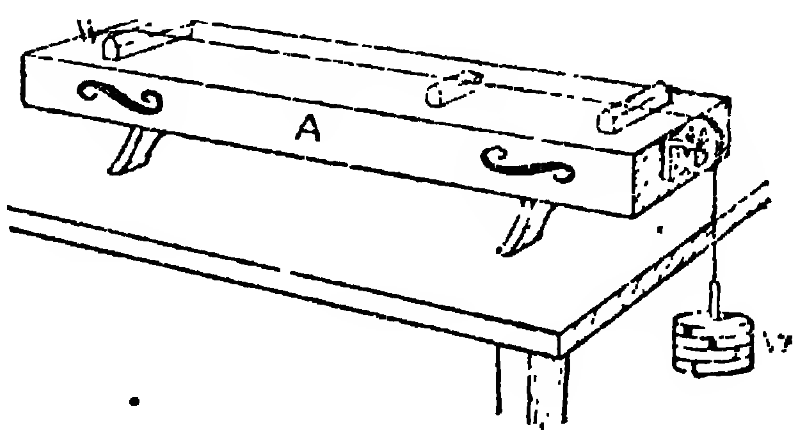

FIG. 16

apparatus is shown in fig. 16. It consists of a sounding

box A,* one end of which is provided with some pegs to fasten the strings. In the horizontal form of the apparatus, a thin wire, one end of which is fastened to a peg passes across the top of the box over two bridges fixed at the ends of the box. The wire then passes over a smooth pulley fixed at the other end of the sounding box and carries a scale pan. There are several movable bridges, by placing any of which under the wire, a certain length of the wire can be set to vibrations.

A vertical form of the sonometer is perhaps more convenient. In this form the sounding-box is suspended vertically from a rigid support. The bridges can be shifted on the sounding-box and fixed at any desired place on the box by screws. To stretch the wire, which passes over a small wheel at its lower end, the load is directly suspended from the wire ; so that the friction due to the pulley is banished and that on the bridges minimised.

It is important that the base on which the string and bridges are mounted should be massive and strong, yielding as little as possible to the forces of tension applied to the string.

When the string is set to vibration, transverse waves are set up in the string which are propagated along the

*Without the sounding-box the string will radiate very little sound energy into the surrounding air. When it is desired, therefore, to employ a vibrating string as a source of sound its vibrations are transferred to another body which is more suitable to transmit vibrations into the air. Using the sounding-box in sonometer, the sounding-board in the piano or the belly in a violin, a much larger vibrating area is brought into contact with air and the rate of radiation of sound . . . tly increased.

string with a velocity given by $\sqrt{\frac{T}{m}}$. These waves on reaching the bridges get reflected and travel backwards. Thus between the bridges there are two sets of identical waves travelling with the same velocity in opposite directions. These give rise to the formation of stationary waves between the bridges having fixed nodes and antinodes.

To verify the law of length, several tuning forks of frequencies n_1, n_2, n_3, etc., are taken. The string of the sonometer is kept stretched by placing a load on the scale pan attached to it.

Two bridges are placed on the box below the wire so as to include a certain length of the wire. A tuning-fork, say of frequency n_1, is taken, it is lightly struck and the position of one of the bridges is adjusted so that the string between them, when set to vibrations, is in unison with the vibrating tuning-fork. The unison may be tested by the ear or by placing a light body at the middle point of the string which will be thrown off as the string will be set to resonant vibration, when the vibrating tuning-fork is pressed upon the sounding-box of the sonometer. Carrying out the experiment with different forks with the *same wire* stretched with the *same tension*, it will be found that if lengths l_1, l_2, l_3 etc. are in unison with tuning-forks of frequencies n_1, n_2, n_3, etc.

$$n_1 l_1 = n_2 l_2 = n_3 l_3 = \text{etc.}$$

This verifies the law of length.

A convenient way of verifying the law of tension is to stretch another wire, called the auxiliary wire, along-side the previous one which may be termed the experimental wire. A certain length of the experimental wire is kept

fixed between two bridges and the wire is stretched with a tension T_1 say. The auxiliary wire is stretched with a fixed tension and the length of its vibrating segment is adjusted until it is in unison with the fixed length of the experimental wire. Let the length of the vibrating segment of the auxiliary wire be l_1. The tension of the experimental wire is changed to T_2, T_3, etc. Due to the change of tension the frequency of its vibrating segment is altered. Without altering the tension of the auxiliary wire, the length of its vibrating segment is adjusted in each case such that the lengths l_2, l_3, etc. are in unison with the fixed length of the experimental wire when its tensions are T_2, T_3, etc. respectively. Let n_1, n_2, n_3, etc. be the frequencies of the fixed length of the experimental wire when its tension are T_1, T_2, T_3 etc.

Since on the experimental wire the tension changes only, we have, if the law of tensions is true

$$n_1 \propto \sqrt{T_1} \text{ and } n_2 \propto \sqrt{T_2}$$

$$\therefore \quad \frac{n_1}{n_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}} \quad \ldots \quad \ldots \quad (1)$$

On the auxiliary wire since the length of the wire changes only, we have by the law of length

$$n_1 \propto \frac{1}{l_1} \text{ and } n_2 \propto \frac{1}{l_2}$$

$$\therefore \quad \frac{n_1}{n_2} = \frac{l_2}{l_1} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Hence if the law of tension is true, we have from (1) and (2)

$$\frac{l_2}{l_1} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$$

$$\text{or } l_1\sqrt{T_1} = l_2\sqrt{T_2} = l_3\sqrt{T_3} = \text{etc.} \quad \ldots \quad (3)$$

By measuring l_1, l_2, l_3, etc. and T_1, T_2, T_3, etc., the relation (3) can be found to be true.

This verifies the law of tension.

The law of mass can be verified in the same way as the law of tension. Keeping the length of the vibrating segment of the experimental wire and the tension stretching it unaltered, the wire is replaced by others having different linear densities. The length of the vibrating segment of the auxiliary wire, stretched with a constant tension, is adjusted in each case to have unison with the same length of the different experimental wires. Suppose, the linear densities of the different experimental wires are m_1, m_2, m_3, etc. and the lengths of the auxiliary wire in unison with them are l_1, l_2, l_3, etc. Let the frequencies of these experimental wires be n_1, n_2, n_3, etc. respectively.

If the law of mass is true.

$$n_1 \propto \frac{1}{\sqrt{m_1}} \text{ and } n_2 \propto \frac{1}{\sqrt{m_2}}$$

$$\therefore \quad \frac{n_1}{n_2} = \frac{\sqrt{m_2}}{\sqrt{m_1}} \quad \ldots \quad \ldots \quad (1)$$

Since the law of length holds good with the auxiliary wire,

$$n_1 \propto \frac{1}{l_1} \text{ and } n_2 \propto \frac{1}{l_2}$$

$$\therefore \quad \frac{n_1}{n_2} = \frac{l_2}{l_1} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Hence if the law of mass is true, we have from (1) and (2)

$$\frac{l_2}{l_1} = \frac{\sqrt{m_2}}{\sqrt{m_1}}$$

$$\text{or} \quad \frac{l_1}{\sqrt{m_1}} = \frac{l_2}{\sqrt{m_2}} = \frac{l_3}{\sqrt{m_3}} = \text{etc.} \quad \ldots \quad (3)$$

By measuring l_1, l_2, l_3, etc. and m_1, m_2, m_3, etc. the relation (3) can be found to be true.

This verifies the law of mass.

71. Melde's Experiment—A very suitable way of verifying the laws of transverse vibration of a stretched string is due to Melde (1859-60). In Melde's experiment a tuning-fork of known frequency is coupled to a string. The string is kept stretched under a tension and is thrown into resonant vibration by the tuning-fork which is kept vibrating by being struck with a hammer or better electrically.* The experiment is usually carried in two ways—(1) in which the line of vibration of the tuning-fork is *perpendicular* to the length of the string, called the **transverse arrangement** and (2) in which the line of vibration of the tuning-fork is *along* the length of the string, called the **longitudinal arrangement.**

(1) *Transverse Arrangement*—The diagram of the transverse arrangement is shown in fig. 17. One end of a

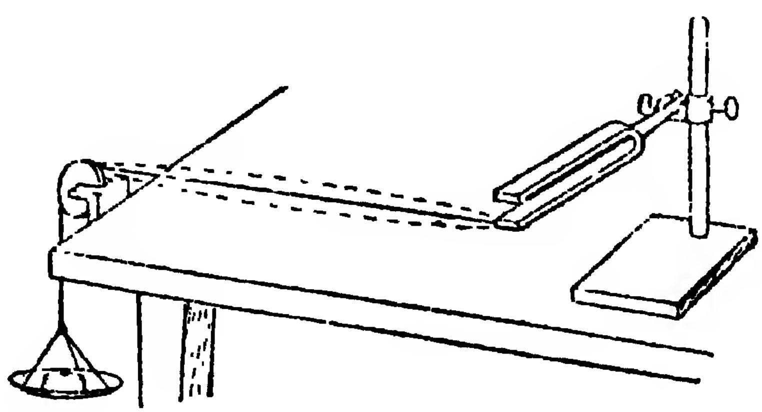

FIG. 17

uniform thread is fastened at one prong of a long massive tuning-fork ; the string passes over a smooth pulley and

* For electrically driven tuning-forks, see Chapter XI.

carries a small scale-pan at its other extremity. The fork is clamped firmly on a massive base in such a manner that the line of vibration of the prongs is at right angles to the thread.

A weight is placed on the scale-pan and the fork is set to vibrations by striking it with a hammer or by a bowing it with a violin bow or electrically.

The thread will be set to forced vibrations. The transverse waves thus set up in the thread will travel along it with a velocity depending on its tension and linear density. The waves will then be reflected from the pulley and travel backwards. Thus in the thread, there will be formed two sets of identical waves travelling in opposite directions with the same velocity. This will give rise to stationary waves in the thread having fixed positions for nodes and antinodes.

In general, due to forced vibration the amplitude of vibration of the thread will be small, the thread having pseudonodes and antinodes (false nodes and antinodes). By properly adjusting the tension and length of the thread, the thread will be set into resonant vibration having the maximum amplitude and prominant nodes and antinodes. When this occurs, the mean distance l between two consecutive nodes is measured by placing two mounted pointers below them. The wave-length of the transverse waves is evidently twice the distance between the consecutive nodes. The tension and mass per unit length of the thread is found out and the frequency of vibration of the thread is calculated on the assumption of the law.

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Since resonance is taking place the frequency of the thread must be equal to the frequency of the fork.* It will be found that the frequency of the fork calculated in this way, on the basis of the law of transverse vibration of a stretched string, agrees fairly with the known frequency of the fork ; hence the law is verified.

(2) *Longitudinal Arrangement*—In the longitudinal arrangement of Melde shown in fig. 18, one end of a uniform thread is fastened to one prong of a long massive tuning-fork. The thread, as before, passes over a smooth pulley and carries a scale-pan at its other end. The fork is clamped such that the line of vibration of the prongs is along the length of the thread. The experiment is conducted by loading the scale-pan and setting the fork into vibration as before. Stationary waves are set up in the thread as in the previous arrangement having fixed positions for nodes and antinodes. The tension and length of the thread are varied until the thread vibrates with the maximum amplitude having prominent nodes and antinodes. The average distance l between two consecutive nodes is measured and the frequency of vibration of the thread is calculated on the assumption of the law of transverse vibration of a stretched string.

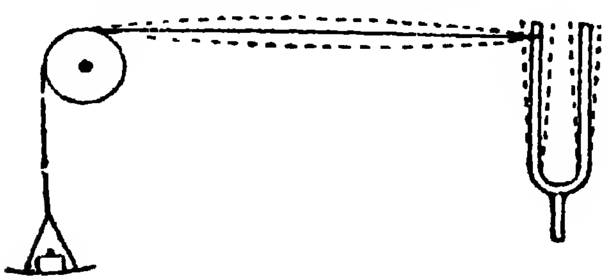

FIG. 18

$$n=\frac{1}{2l}\cdot\sqrt{\frac{T}{m}}$$

But the frequency of a vibration of the fork in this case is *not* equal to that of the thread. This is a curious and

*See Art. 89.

somewhat unusual character of maintenance in which the frequency of vibration of the driver, *viz*. the fork is *double* the frequency of vibration of the driven,* *viz* the thread. Lord Rayleigh has explained this by considering that the vibratory motion of the prong of the tuning-fork renders the tension of the thread periodically variable. The full mathematical treatment of Rayleigh is beyond the scope of the book † An easier explanation of the phenomenon is given below.

Let the thread be horizontal and the prong to which the thread is attached move between A and B (fig. 19, a) in

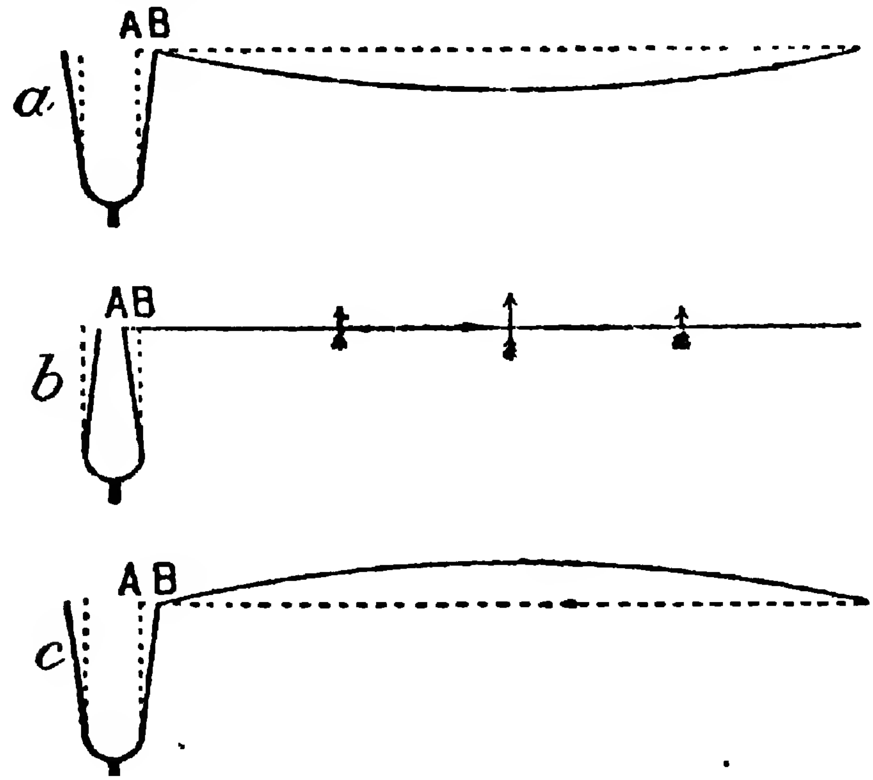

FIG. 19

course of its vibratory motion. As the prong moves from A to B, the thread becomes slack and gets fully displaced as

* See Art, 94

† For the treatment of Rayleigh, see "The Theory of Sound," Lord Rayleigh. vol, 1, page 82 or 'A Text-Book of Sound.' Barton, Page 880.

shown in fig. 19, *a*. As the prong moves back from B to A the thread is tightened and takes up its horizontal position as shown in fig. 19, *b*. The thread is in its motion upwards. During the next vibration as the prong moves from A to B, the thread becomes slack again but the thread due to its previous inertia of upward motion, is now displaced in the upward direction as shown in fig. 19, *c*. Again during the backward journey of the prong from B to A the thread is tightened as in fig. 19, *b*, but this time the thread is in motion downwards. Thus during the time when the fork makes two vibrations, the thread makes one vibration. In other words, the frequency of vibration of the fork is double the frequency of vibration of the thread.*

The frequency of vibration of the thread is calculated on the assumption of the law of transverse vibration of a stretched string, by measuring its tension and mass per unit length. The frequency of vibration of the fork is found out by doubling it. It will be found that the frequency of the fork calculated in this way agrees well with the known frequency of the fork, which verifies the law.

72. Transverse Vibration of Bars—If the ratio of the diameter to the length of a wire be more and more increased the stiffness or rigidity of the wire becomes more and more important in providing forces of restitution to its transverse vibrations, rather than the tension of the wire. A rod (of circular section) or a bar (of rectangular section) in acoustics is said to be *ideal*, the transverse vibrations of which are affected by stiffness *only* and unaffected by tension.

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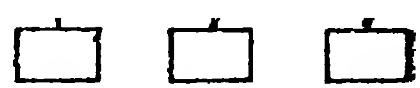
* Professor Raman has shown that the note of the string may be any sub-harmonic of that of the fork. See Phil. Mag. 24, 518, 1912.

The mathematical theory of the transverse vibrations of a rod or bar is beyond the scope of the book. For this, a higher text-book, such as Rayleigh's Theory of Sound or Barton's Text-book of Sound or Lamb's Dynamical Theory of Sound may be consulted. In the following is given an elementary notion of the transverse vibrations of an 'ideal bar.'

There are several modes of transverse vibration of a rod or bar according to the way in which it is supported. If the bar is clamped at one end and free at the other, it is called a *fixed free* bar, if both the ends rest on suitable supports, it is called a *free-free* bar. Lastly, if it is clamped at both ends, it is called a *fixed-fixed* bar.

73. **Fixed-Free Bar**—The bar is clamped firmly in a vice at one of its ends and displaced laterally at the other. The first three modes of vibration are shown in figs. 20 (*a*), (*b*) and (*c*). Out of these (*a*) is the fundamental form and (*b*) and (*c*) are the first two overtones. In all the modes there is an antinode at the free end and a node at the fixed end. In the form (*b*), there is one intermediate node and one intermediate antinode. In (*c*), there are two intermediate nodes and two intermediate antinodes. In appearance the different modes of vibrations resemble the longitudinal vibrations of the air column in an organ pipe closed at one end*. But the difference is that in an organ pipe emitting



a *b* *c*

FIG. 20

*See Art. 140

the first overtone, the intermediate node is situated at a distance $l/3$ from the open end, where l is the length of the organ pipe. In the transverse vibration of a bar emitting the first overtone, (fig. 20 b) the intermediate node is situated at a distance $\cdot 2261 l$ from the free end, where l = the length of the bar. This makes the overtone of the organ pipe, the *second harmonic* of the fundamental tone. But in the case of the bar, the overtone is *not a harmonic* of the fundamental. Similarly, in the vibrations of the air column in a closed organ pipe, resembling fig. 20c, the two intermediate nodes are at distances $\frac{l}{5}$ and $\frac{3}{5}l$ from the open end, while in the transverse vibrations of a bar represented by fig. 20c, the intermediate nodes are respectively at distances $\cdot 1321 l$ and $\cdot 4999 l$ from the free end. This makes the note emitted by the organ pipe, *fourth harmonic* of the fundamental tone, but the note emitted by the bar is *not a harmonic* of the fundamental. In general, the various overtones emitted by the transverse vibrations of a bar are not harmonics of the fundamental.

74. Free-Free Bar—If a bar of length l, free at both ends, vibrates in the fundamental form (fig. 21a), the nodes occur at distances $\cdot 2242 l$ from either of the free ends. The bar is supported at the nodal positions on little pads of india-rubber and excited by an ordinary bow.* When the bar vibrates so as to emit its first overtone (fig. 21b), the distance of either of the two extreme nodal

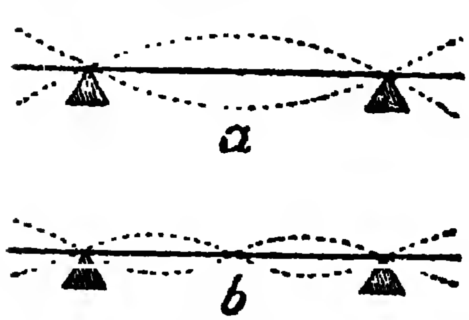


FIG. 21

*See 'Lateral Vibrations of Bars', Phil. Mag., Nov. 1907.

positions from the neighbouring end of the bar is ·1321l and the middle node is at the middle point of the bar. The overtone produced is not a harmonic of the fundamental.

75. Fixed-Fixed Bar—The bar is clamped by means of vices at both ends and set to vibration by an ordinary bow. The same series of tones as in the case of a free-free bar, are produced.

In general, the frequency of transverse vibration of a bar is inversely proportional to the square of the length of the bar, directly as the thickness of the bar, and the frequency is proportional to the velocity of longitudinal waves through the bar.

76. Tuning-Fork—The early development of the tuning-fork is mainly due to König. He made tuning-forks having a wide range of frequencies. In recent times, tuning-forks are growing in importance as standards of frequency in acoustical determinations. In the electrically driven form[*] they serve to control electrical circuits so as to form standards of electrical frequency of great accuracy.

Forks are now commonly used as standards of time. The period of vibration of a tuning-fork is very constant and serves as a convenient standard for measuring time intervals with accuracy.

The sound emitted by a properly made tuning-fork, when bowed gently, is practically a pure tone. For, it is comparatively difficult to obtain the overtones and the overtones, if present, die away very quickly compared to the fundamental tone : so that a few seconds after the fork is struck, its sound is almost pure. If the fork is mounted on its resonance box, its sound is made still purer ; the box has a series of natural tones, but none of those lie near the

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* See Art. 98.

tone of the fork except the fundamental, which is strengthened thereby.

Different writers look upon the tuning-fork in different manners. According to the earliest investigators, such as Chladni, a tuning-fork may be supposed to be developed by bending a free-free bar at its ends so as to take the shape of the letter U. If a free-free bar vibrating in the manner shown in fig. 21, (*a*) is gradually bent at the middle, the nodes approach each other as shown in fig. 22 (*b*), (*c*), (*d*) and (*e*).

When the two limbs are parallel, the arrangement forms

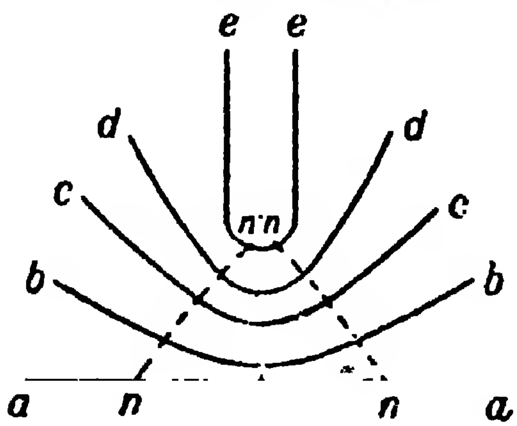

FIG. 22

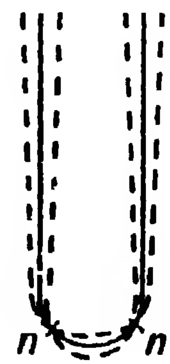

FIG. 23

a tuning-fork. The nodes are very near to each other, and consequently the amplitude of vibration at the antinode situated at the middle of the fork is very small compared to that at the ends of the two limbs which are called the *prongs*. The addition of a stem between the two nodes acts in two ways. It adds to the mass and stiffness at the antinode. This reduces the amplitude of vibration of the middle antinode, but this small vibration of the stem (fig. 23) serves to actuate a resonance box when the stem of a vibrating tuning-fork is pressed upon it.

This method of consideration of a tuning-fork shows that when a tuning-fork vibrates, the two prongs alternately

approach and recede from each other, for when the straight bar vibrates in the manner shown in fig. 21 (*a*) the two ends move up or move down together which continues in the bent bar.

Lord Rayleigh considers a tuning-fork as made up of two symmetrical fixed-free bars joined together at their fixed ends and attached to a block of metal. The tuning-fork is thus regarded as a double source of vibrations and the small vibrations of the stem are explained from a different standpoint.[*]

77. **Frequency of Tuning-fork**—To a near approximation, each prong of the tuning-fork may be regarded as a fixed-free bar although the vibration frequency of the tuning-fork is complicated by the presence of an additional block at the centre of the bend and the stem attached thereto. On this basis, the frequency of (fundamental) vibration of the tuning-fork is given by

$$N=\frac{m^2 k}{2\pi l^2}\sqrt{\frac{Y}{\rho}}$$

where $k=\dfrac{a}{\sqrt{12}}$, a being the thickness of the prong,

$l=$ the length of the prong,

Y and ρ are the Young's modulus and density of the material of the fork respectively,

and $m=1{\cdot}875$ (a numerical constant).

For steel, $\sqrt{\dfrac{Y}{\rho}}=523700$ cms./sec.

whence $N=84590\dfrac{a}{l^2}$ nearly.

Thus the frequency of vibration of a tuning-fork varies directly as the thickness of the prongs and inversely as the

* See 'Theory of Sound,' Rayleigh, vol. 1, page 58 or A Text-Book of Sound,' Wood, page 119.

† See 'A Text-Book of Sound,' Barton, page 298.

square of the length of the prong. It also varies directly as the velocity of longitudinal waves in the material of the fork. The frequency is independent of the width of the prong.

78. Variation of the Frequency of a Tuning-Fork with Temperature—A change in temperature of the tuning-fork alters the thickness and length of its prongs and also the Young's modulus and the density of the material of the fork and necessarily changes its frequency. Thus if N represents the frequency at a lower temperature and N_t that when the temperature is raised through $t°C$, we have from the previous article

$$N = \frac{m^2}{2\pi\sqrt{12}} \frac{a}{l^2} \sqrt{\frac{Y}{\rho}} \quad \ldots \quad \ldots \quad (1)$$

and $$N_t = \frac{m^2}{2\pi\sqrt{12}} \frac{a(1+\alpha t)}{\{l(1+\alpha t)\}^2} \sqrt{\frac{Y(1-\beta t)}{\rho(1-3\alpha t)}}$$

where α = the coefficient of linear expansion of material of the fork and β = the coefficient of decrease of Young's modulus with rise of temperature.*

$$\therefore \quad N_t = \frac{m^2}{2\pi\sqrt{12}} \frac{a}{l^2} \sqrt{\frac{Y}{\rho}} \cdot \frac{1+\alpha t}{1+2\alpha t} \cdot \frac{1-\frac{1}{2}\beta t}{1-\frac{3}{2}\alpha t}$$

(neglecting the higher powers of α and β compared to the first)

$$\text{Or } N_t = N(1+\tfrac{1}{2}\alpha t)(1-\tfrac{1}{2}\beta t)$$

$$= N\left(1 - \frac{\beta-\alpha}{2} t\right)$$

For steel, $\alpha = \cdot000012$ nearly and $\beta = \cdot00024$ †

$$\therefore \quad N_t = N(1 - \cdot000114t)$$

Hence with rise of temperature, the frequency of tuning-fork decreases. The coefficient of decrease with increase of temperature is ·000114. König found experimentally that the temperature coefficient is ·000112.

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* With rise of temperature, the elastic constants decrease in general. See Author's 'Text-Book of General Physics,' Art. 116.

† Taken from Kaye and Laby's Table of Physicial Constants.

79. Transverse Vibration of Plates—The experimental study of the vibrations of plates originates with Chladni (1787)*. Chladni used glass plates of various symmetrical shapes. The plates were bored through their centres of symmetry and fixed to a vertical upright at the centre by a screw and clamp. Very fine sand was scattered evenly over the plate and the plate was excited by a bow. By holding the plate between the fingers, at one or more points near its edge, the transverse vibrations of the plate were damped at these places. The sand particles were displaced and collected at the nodal lines. The distribution of the nodal lines on the plate is known as Chladni's figure. Fig. 24 represents some of Chladni's figures with square plates. To obtain fig. (1) the square plate is held at the middle of

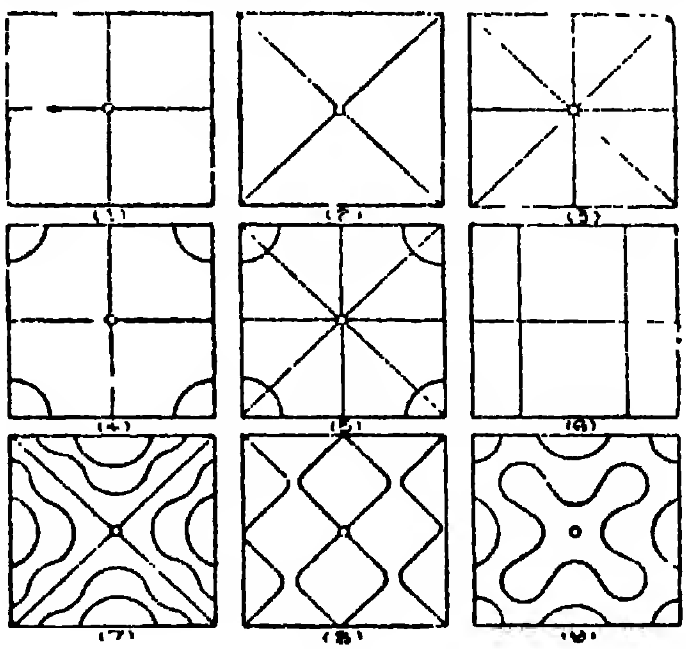

FIG. 24

one side and bowed at a corner. The nodal lines consist of two straight lines parallel to the sides of the plate passing

* For a theoretical study of the subject see "Theory of Sound" vol. 1, Rayleigh or "The Dynamical Theory of Sound", Lamb.

through its centre, dividing the plate into four segments. It is evident that when one segment moves upwards during its vibration, the adjacent segment moves downwards in order that the separating line between them may be a nodal line.

Fig. (2) is obtained by bowing the plate at the middle of one side and holding the plate at one of its corners. Chladni gave 52 figures with a square plate 45 circular, 30 hexagonal, 26 elliptical, etc.*

Savart found that if instead of sand, fine powder, such as lycopodium powder, was used to obtain Chladni's figures, these powders collected at the antinodes instead of at the nodes. Faraday† showed that the effect was due to the vibrations of the air. In vacuum the powders collect along the nodal lines.

Circular plates fixed at the centre give two classes of nodal lines,—(1) radial lines dividing the plate into an even number of sectors and (2) circles concentric with the plate.

80. Transverse Vibration of Membranes—A theoretical membrane is a perfectly flexible and uniform solid lamina of infinitesimal thickness which is stretched on all sides by a tension. For the mathematical treatment‡ of the lamina, it is supposed that the tension is unaltered by the vibrations of the membrane as in the case of stretched strings. The various overtones emitted by a membrane do not form a harmonic series. The kettledrum, in which the membrane is used to produce the sound, is therefore regarded as an instrument to mark the rhythm rather than

* For an experimental study of a large number of modes of vibrations see 'Tyndall's Sound', page 148.

† Phil, Trans., page 299, 1831.

‡ See 'Theory of Sound' Lord Rayleigh or The Dynamical 'Theory of Sound,' Lamb.

to produce a musical note, as the note emitted by it is unmusical in character owing to the presence of inharmonic overtones.

If, however, the mass per unit area of the membrane gradually decreases from the centre to the peripherical parts. the notes produced by it form a harmonic series. This principle is adopted in the Indian musical instrument *Tabla* where by artificial loading, the mass per unit area is made to decrease from the centre to the edges.

81. Vibration of Bells—A large variety of bells are used for different purposes. But practically in all cases the bell is supported at its centre of symmetry and excited by striking near the edge. It may be regarded as a progressive development of a curved circular plate or a cylindrical shell with one end closed. The vibrations of a bell are somewhat similar to those of circular plates. The nodal lines are of two classes :—(1) *radial lines* extending from the point of support to the edge and dividing the bell into an even number of equal sections, these radial lines being called *nodal meridians* ; and (2) *nodal circles* across the bell at various distances from the support.

Fig. 25 shows the simplest mode of vibrations of a bell. The nodal meridians pass through four points N_1, N_2, N_3, and N_4. The bell is thus divided into four segments. During vibrations when one segment moves outwards, its adjacent segment moves inwards. Thus the adjacent segments are in opposite phases of vibration. The various overtones emitted by a bell do not form a harmonic series.

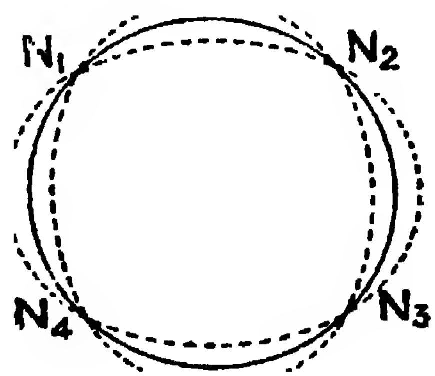

FIG. 25

Lord Rayleigh made systematic experimental investigasion of the vibrations of the bell.*

Examples

1. Describe the experiment of Melde's which has for its purpose the illustration of the laws governing the vibration of strings.

The tuning-fork makes 198 vibrations per second; a mass of one kilogramme is placed in the scale pan, the wire is of platinum of sp. gr. 21, the diameter is ·5 m.m.

What must be the effective length of the wire for a node to be formed at its middle? *(C. U. 1910)*

2. In what essential point do transverse vibrations of a rod differ from a transverse vibration of a string?

Comparing a vibrating tuning-fork with a vibrating rod supported in a certain way, show by a diagram that the ends of a tuning-fork alternately approach and recede from each other, and the attachment of the stem does not interfere with the vibrations.

Show also how the vibration frequency of a tuning-fork can be accurately determined. *(C. U. 1911)*

3. Describe accurately how you can determine the velocity of sound along a stretched wire in the laboratory. *(C. U. 1917)*

4. Describe Melde's experiment on vibration of a string.

A fork vibrates along the length of a string that is attached to one prong, and is stretched by a load of 41·8 grammes. If the length of the string be 20 cms. and its mass per unit length be 0·085 gramme, and if the string vibrate in 4 segments, calculate the frequency of the fork. *(C. U. 1919)*

5. Describe any method for finding experimentally the velocity of propagation of transverse waves along a stretched string. Does the velocity depend on the tension?

Indicate the variation of the velocity with the tension, if there be any, by a graph. *(C. U. 1920)*

6. Describe any one form of Melde's experiments, and indicate how the laws of vibrations of a stretched string can be verified with Melde's apparatus. *(C. U. 1923)*

*See 'Theory of Sound,' Rayleigh, Vol, 1, page 285.

7. Explain the production of nodes and loops in the case of strings. (*C. U. 1925*)

8. Explain with diagrams the nature of the vibrations of a tuning-fork. What special features make it a valuable instrument in the scientific study of sound ? (*C. U. 1931*)

9. Prove that the frequency of vibration of a stretched string is equal to $\frac{1}{2l}\sqrt{\frac{T}{m}}$, where T is the tension, l the length and m the mass per unit length of the string. Explain what harmonic will be present and what absent when the string is struck at the middle point. (*C. U. 1932*)

10. Obtain an expression for the velocity of transverse waves in a string.

Explain why the pitch of a string, vibrating transversely, depends on the tension, while that of a string vibrating longitudinally is not greatly affected by change of tension. (*C. U. 1939*)

11. Calculate the velocity with which a transverse disturbance runs along a stretched string. Explain how in case of a bowed string, the higher harmonics may be obtained. (*C. U. 1941*)

12. State the laws which govern the vibration of strings. One end of a stretched string is fastened to one of the prongs of a vibrating tuning-fork ; compare the vibrations excited in the string when the prong is moving parallel to the string with those when the prong moves transversely. (*C. U. 1943*)

13. Distinguish clearly between overtones and harmonics. Give a few instances of overtones that are not harmonics.

What are the functions of harmonics in a musical note and how can you detect their presence ? (*C. U. 1944*)

14. Explain with diagrams the nature of the vibrations of a tuning-fork. For what special features, it is a valuable instrument for study of sound ? (*C. U. 1947*)

15. Enumerate the laws of vibrations of a stretched string. Wires of equal length of brass and steel are stretched on a sonometer and adjusted to emit the same fundamental note. If the tensions in the two cases are 5 and 8 kilograms weight respectively and the diameter of the steel wire is 0·8 mm find that of the brass wire ; the densities for brass and steel being 8·4 and 7·8 respectively. (*C. U. 1948*)

16. Obtain an expression for the velocity of transverse waves in a string ; prove that the frequency of vibrations of a streched string is equal to $\frac{1}{2l}\sqrt{\frac{T}{m}}$.

Explain the symbols used. Indicate what harmonics will be present and what absent when the string is struck at the middle point. (*C. U. 1952*)

CHAPTER X

REFLECTION, REFRACTION AND DIFFRACTION OF SOUND

82. Reflection of Sound—When sound waves meet an obstacle or the surface of separation between two different media they are reflected. The side of a building, a row of trees, the side of a hill, etc., serve as obstacles for the reflection of sound waves. Sound waves travelling in ordinary air are also reflected when they meet the moist air on a river, as moist air has a lower density and plays the function of a medium different from comparatively dry air.

The laws of reflection of sound are the same as those of reflection of light,—*viz. that the angles of incidence and reflection are the same and that they are in the same plane.* The only apparent difference between light and sound reflection is that light waves are found to be reflective from small polished surfaces, while, for the reflection of sound waves large extended surfaces are necessary. The reason for this is to be found in relative wave-lengths of light and sound.

For the reflection of any wave from an obstacle, the linear dimensions of the obstacle must be large compared to the length of the wave. The wave-length of light is small and a small obstacle is sufficient for the reflection of light waves. On the other hand, the wave-length of sound is, in general, large and a comparatively large obstacle is necessary for the reflection of sound waves.

Huyghen's principles of secondary wavelets account for the reflection and refraction of light waves. These hold good in the case of sound waves also.*

83. Experiments on Reflection of Sound—(1) Arrange a large plane cardboard A (fig. 26) vertically and place a

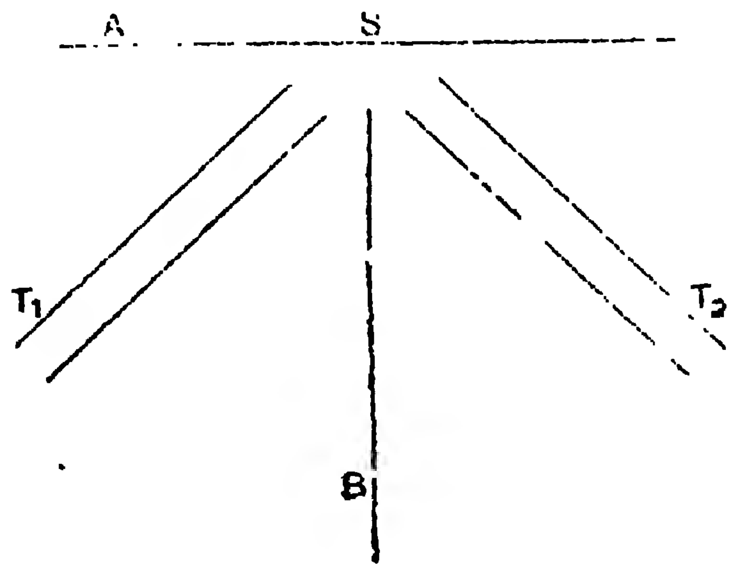

FIG. 26

long tube T_1 horizontally with its axis pointing at some point S in the cardboard. Hold a small watch near the end of the tube. If a second long tube T_2 is held with its axis pointed towards S, and the ear placed at its end, it will be found that the sound from the watch is distinctly heard when the axes of the tubes are equally inclined to the normal to the cardboard at S and when the two axes and the normal to the cardboard at S are in the same plane. Another cardboard B is placed vertically to cut off the direct sound.

(2) Arrange two large concave spherical surfaces A and B co-axially (fig. 27) and hold a small watch at the principal

* For Huyghens' principles a standard text-book of Physical Optics, such as Preston's Theory of Light, is to be consulted.

focus of one of the surfaces *A*. The sound of the watch will be reflected from the concave surface *A* obeying the optical laws of reflection and fall upon the second surface *B*. Reflected from the second surface the sound will converge at the principal focus of it. To detect the reflect sound, take a funnel connected to an india-rubber tube, place the end of the tube to the ear and try for the sound image by the funnel. It will be found that the sound of the

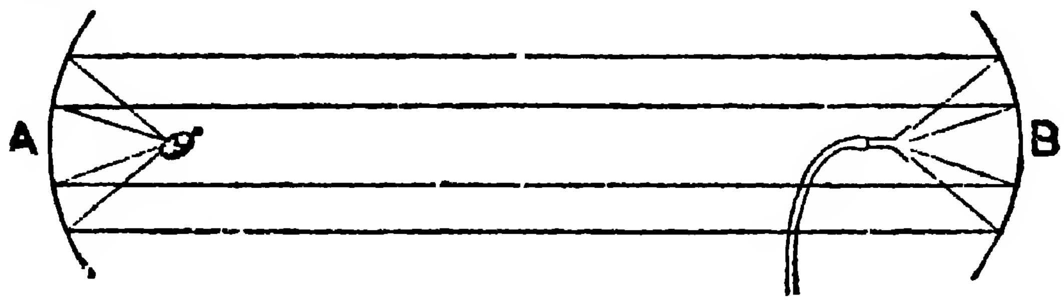

FIG. 27

watch will be prominently heard when the funnel is at the principal focus of the surface *B*.

84. Echoes—The repetition of a sound produced by reflection at an obstacle, such as the wall of a building or a cliff, is called its echo.* In order that the echo of a sound may be appreciated by the ear, the reflected sound must reach the observer *at least* one-tenth of a second later than the direct sound. A clear echo is produced by a sharp sound of impulsive nature; a low pitched sound hardly produces a perceptible echo.

The 'echo' finds an important application in measuring the depth of ocean. The old method known as the 'wire method' for measuring the depth of ocean necessitated a great reduction of speed of the ship and involved a long

* Many interesting experiments on 'echo' are described in Tyndall s Sound.

time. The method of echo supersedes the wire method and can be applied with moderate speed of the ship, and the operation requires a few seconds only. Echo-sounding systems have been devised in America, France, Germany and Great Britain. The systems differ in the source of the sound and in receiving the echoes. The principle consists in measuring the time taken by sound to travel from the surface to the bottom and the echo produced to reach the surface. From a knowledge of the velocity of sound under the condition of salinity and temperature of the water, the depth of the ocean is calculated.

The principles of echo have also been applied for detecting at a distance of any rocks, icebergs or wrecks. The method consists in sending a beam of sound waves from a ship in its direction of motion and detecting any echo produced by reflection of the sound waves by means of a 'hydrophone'. The hydrophone contains a granular-carbon microphone resembling the inside of a telephone transmitter. When the reflected sound waves fall on a diaphragm in front of the microphone, the diaphragm and with it the microphone are set to vibrations which can be detected from the ship.

Recently, the echo method has been employed with success to determine the depths of geological strata containing valuable minerals or oil beneath the earth's surface. Rankine (1929) describes a method in which an intense sound is made on the surface of the earth by a large explosion. The sound wave due to the explosion is received on sensitive seismographs placed at different distances. The sound of explosion travels along different paths, *viz.* the direct path through the surface layer of the earth, the path

after reflection from the surface of the mineral strata, etc. and reaches the seismograph at different times, from which the depth of the mineral strata is found out.

85. Harmonic Echo—The phenomenon of reflection plays an important part in the acoustics of building. The dimensions of a reflector, as has been already mentioned, in Art. 82, is a prominent factor in reflection. If a note, which is a mixture of several sounds, falls upon a reflector, the reflector has a property of reflecting certain wave-lengths better than others depending upon its dimensions. If the dimensions of a reflector be small, it reflects waves of larger frequencies better than waves of smaller ones. Thus, an echo is not always an exact reproduction of the original. Rayleigh in 1873 noticed cases in which the upper components of a complex note predominated in the echo. These were called 'harmonic echoes' by him as the pitch of the reflected sound was apparently increased.

86. Musical Echo—If the reflector consists of a large number of obstacles placed at regular intervals, such as a wooden fence, the palings of which are spaced at equal distances in echelon form, the echo consists of a number of reproduction of the original sound reaching the ear at regular intervals of time. For, the sound wave striking each paling and being reflected there has to travel a little further and consequently reaches the listener's ear a little later than the wave which struck the preceding paling in the row. A clap of hand is thus reflected as a succession of claps at regular intervals and is perceived by the ear as a short musical echo.

87. Whispering Gallery—An interesting case of reflection of sound is observed in the well-known whispering

gallery of St. Paul's Cathedral in London. The gallery has a circular form running round the base of the inside of a hemispherical dome. If a person whispers along the wall, on one side, he can be distinctly heard at any other part of the gallery provided the listener places his ear near to the wall, but the whisper remains inaudible at any point far from the wall. According to Airy, the effect is to be observed at the point of the gallery diametrically opposite to the source of sound. According to the theory given by Rayleigh a whisper is audible with abnormal loudness not only at the point diametrically opposite to the whisperer but the whisper seems to creep round the gallery horizontally along that arc towards which the whisperer faces. Raman and Sutherland made experiments in 1921 with a high pitched source and a sensitive flame and their observations confirm Rayleigh's conclusions. A complete explanation of the phenomenon seems yet to be found.

88. Phase Change on Reflection—

(a) *Reflection at a rigid wall*—We shall study the peculiarities of reflection of longitudinal waves at a surface which is rigid enough not to yield to the alternate compressions and rarefactions inherent in longitudinal waves. The side of a building, the closed end of an organ pipe, etc., approximate to such a rigid wall sufficiently. If a pulse of compressed air reaches such a rigid wall, the motion of the air particles are checked. The effect is comparable to a gas enclosed in a vessel fitted to a piston. If the piston is pushed to the closed end of the vessel so as to compress the gas and then released, the compressed gaseous particles in contact with the closed end will momentarily be at rest due

to inertia but this state of affairs cannot remain long and the compressed region will begin to expand and thereby transmit its state of compression in the backward direction.

In the reflection of sound waves from a rigid wall, a similar state of affairs takes place. A pulse of compression is reflected as a pulse of compression and similarly, a pulse of rarefaction will be reflected as a pulse of rarefaction. Thus a wave train, when it meets a rigid wall, is reflected from it as a similar wave train. *In other words, when reflection takes place from a rigid wall there takes place no phase change between the incident and reflected waves.*

(*b*) *Reflection at a yielding wall*—When a wave of any kind reaches a discontinuity of any type, it undergoes reflection to some extent. Thus, when the sound waves, travelling in a pipe, meet the open end of it, they are reflected. If a wave of compression meets a wall which perfectly yields to the wave, just as a compressed wave travelling along a tube meets its open end, there is much greater freedom for the wave, to expand outwards. For, within the tube a layer of air followed by a compressed layer behind, might move in the forward direction alone, but reaching the open end, the air can expand in the forward direction as well as side ways. The result is that on reaching the open end the compressed layer becomes rarefied and a wave of rarefaction travels backwards. Similarly, a wave of rarefaction on reaching a yielding wall is reflected as a wave of compression. In other words, *when reflections occur at a yielding wall the reflection takes place with a phase change such that the phase of the reflected wave is opposite to that of the incident wave.*

It must, however, be noted that when reflection occurs from a perfectly rigid wall, the reflection is complete but if the reflection occurs from a yielding wall, the reflection is only partial.

89. Refraction—When sound waves pass from one medium to another in which the velocity of propagation of sound waves is different from that in the former, the waves are bent or refracted in a manner somewhat analogous to that of light waves. If the velocity of propagation in the first medium has a higher value than that in the second medium, the waves on entering the second medium from the first are bent in the second medium towards the normal drawn to the surface of separation between the two media at the point of incidence.

The laws of refraction of sound are similar to those of light,—*viz. the sine of the angle of the incidence bears a constant ratio to the sine of the angle of refraction depending only on the nature of the two media and that the angles of incidence and refraction lie in the same plane.*

It has already been stated that Huyghens' principles of secondary wavelets account for refraction of light and sound waves.

90. Experiments on Refraction of Sound—Construct a lens-shaped hollow* vessel such as india-rubber balloon. Fill it with a gas which is denser than air, such as carbon dioxide. Place a small watch on one side of it at a suitable distance. The sound of the watch is distinctly

*Lenses of solid material or of liquid in rubber vessels are unsatisfactory ; for, the sound waves are almost completely reflected from the surface on which they are incident.

heard at a point on the other side of it (fig. 28). The places where the watch is placed and where the refracted sound is heard represent the *acoustic conjugate foci.*

The balloon behaves as an acoustic converging lens.

If the balloon is filled with a gas which is less dense than air, such as hydrogen or coal gas, the waves diverge more after passing through the lens. The balloon behaves as an acoustic diverging lens (fig. 29).

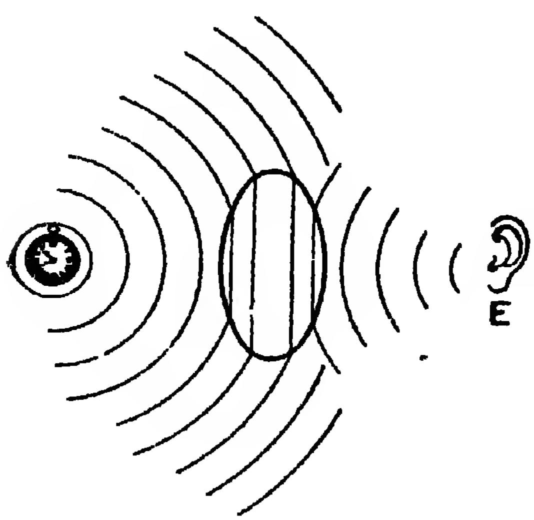

FIG. 28

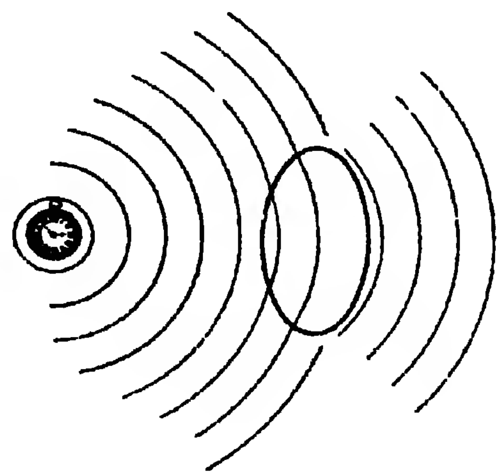

FIG. 29

Sondhauss demonostrated the refraction of sound waves through prisms containing various gases and determined their *acoustic refractive index* relative to air. If μ represents the acoustic refractive index of a gas relative to air, μ can be defined from the optical analogy as

$$\mu = \frac{\text{velocity of sound in air}}{\text{velocity of sound in the gas}}$$

91. Total Internal Reflection : Critical angle—The optical phenomenon of total internal reflection has its ana logy in acoustics. Using the anolagous terms of a ray of light, when a *ray of sound** travelling in a medium meets

* See Art. 26.

the plane of separation with a second medium in which the velocity of sound is greater than that in the first medium, the ray is bent in the second medium away from the normal to the plane of separation. When the ray is incident at such an angle that the refracted ray 'grazes' the surface of separation, the incident angle is known as the *critical angle*. If the incidence takes place at an angle greater than the critical angle, the sound energy is totally reflected without entering the second medium. This phenomenon is known as the *total internal reflection*.

The velocity of sound in air at ordinary temperature is 340 metres per sec. and that in water is 1440 metres per sec. When sound rays travel from air to water, they deviate away from the normal.[*] If C represent the critical angle at the air-water surface.

$$\frac{\sin C}{\sin \frac{\pi}{2}} = \frac{340}{1440}$$

$$\text{or } \sin C = \frac{340}{1440}, \text{ whence } C = 13^{\circ}\cdot 5 \text{ nearly.}$$

Thus sound rays propagating in air, when incident at an angle greater than $13^{\circ}\cdot 5$ at the air-water interface, the rays are reflected within the air. This explains why the voices of bathers whose heads are naturally a little above the water surface, can be plainly heard on the shore.

92. Atmospheric Refraction—

(*a*) *Refraction by Wind*—It is a common observation that sound is more clearly heard with the wind than against it.

Compare the corresponding optical case.

The whistling sound of a distant locomotive is more *intensely* heard when the wind blows from the locomotive to the listener than when the wind blows in the reverse way or when the air is still. The explanation of this was given by Stokes in the following manner.

Let a plane wave front A_0B_0 (fig. 30) from the source, perpendicular to the ground, pass through the successive positions A_1B_1, A_2B_2, etc., after equal intervals of time in still air. If the wind blows from the source to the listener

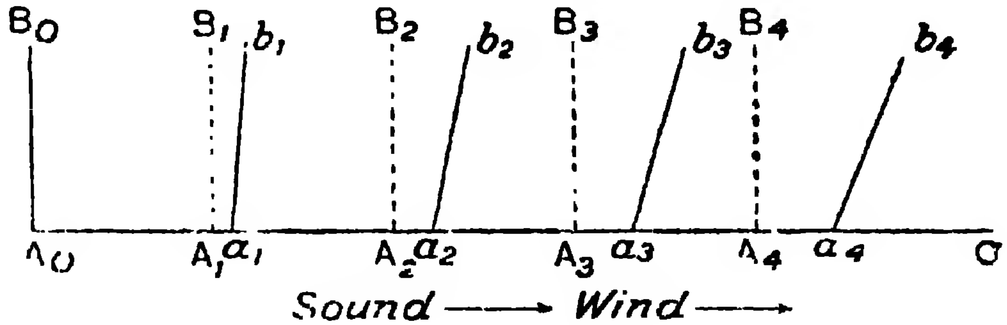

FIG. 30

the velocity of the wind is added to that of the sound. Moreover, the upper layers of air will possess a larger velocity than the lower ones. The result is that the upper parts of the wave front will travel more quickly than the lower parts. Hence, the wave front in its successive positions will turn downwards, such as a_1b_1, a_2b_2, etc. instead of remaining vertical. The rays of sound, which are normal to the wave fronts,* therefore turn towards the ground and a listener on the ground at O has a better chance of hearing.

If, on the other hand, the wind blows from the listener to the source, the velocity of the wind must be subtracted from that of the sound. Moreover, the upper layers of air will possess a larger velocity than the lower ones. Hence

* See Art, 26.

the upper parts of the wave front will travel less quickly than the lower parts. The wave front in its successive positions will turn upwards such as a_1b_1, a_2b_2, etc., (fig. 31)

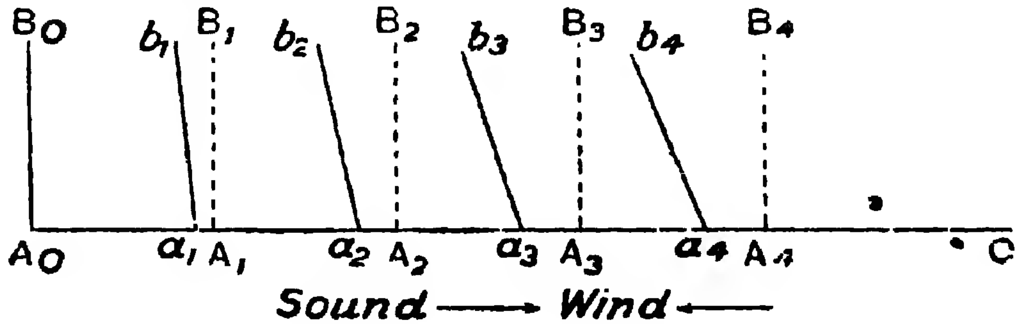

FIG. 31

instead of remaining vertical. The rays of sound which are normal to the wave fronts, therefore, turn upwards and tend to go into the upper air and are thinned near the surface, so that a listener on the ground at O has a less chance of hearing.

(*b*) *Refraction by Temperature Gradients*—Probably every one is familiar with the fact that voices are more clearly heard from a distance at dusk than during the day time. In the day time, there is a fall of temperature at higher levels. Therefore, the velocity of sound is the greatest near the ground and it decreases upwards. The plane wave fronts are thus bent upwards as in fig. 31, which, in absence of such temperature variation, would have been vertical. The effect is similar to the propagation of sound *against* wind and the waves are thinned near the ground so that distant voice is not heard well.

At dusk, the air below cools more rapidly than the air above, so that there is gradual rise of temperature at higher levels. This is often associated with the formation of mist near the earth's surface, specially in the vicinity of seas and rivers. The velocity of sound is least near the ground

and it increases upwards. The plane wave fronts thus turn downwards as shown in fig. 30, which, in absence of such temperature variation, would remain vertical. The effect is similar to the propagation of sound *with* the wind and the waves thicken near the ground so that a distant voice is heard clearly.

93. Diffraction of Sound : Acoustic Shadow—It is commonly observed that sound bends appreciably round any obstacle while the propagation of light is sensibly linear. It has already been referred to, that experiments with very fine obstacles, such as the end of a needle, the edge of a razor blade, etc., show that light also bends round suitable obstacles.

The region behind an obstacle held before a sound source, when the sound is absent, is called, from optical analogy, the *acoustic shadow* of the obstacle. It is generally found that the optical shadow of an obstacle is sharply well defined and hence it is called a '*geometrical shadow*', but the acoustic shadow of an obstacle is not well defined The bending of waves (of light or sound) round obstacles and their encroachment within the geometrical shadow of the obstacle is known as *diffraction*. The reason as to why acoustic shadows are not so well-defined as the optical shadows, is to be found in their relative wave-lengths. In general, the wave-length of sound is of the order of a million times the wave-length of light and the encroachment of waves (of light or sound) within the geometrical shadow increases with the length of the waves. For this, the acoustic shadows are not so well defined as the optical shadows. To obtain sharp acoustic shadows with obstacles of moderate size it is, therefore, necessary to use waves

having short wave-length, *i.e.* the waves which are produced by sources of high frequency.

Sound waves can be diffracted by *acoustic gratings* in the same way as light waves are diffracted. Altberg in 1907 constructed a grating of glass rods about one centimetre apart. As a source of sound, he used the spark discharge of a condenser which possesses the property of generating short sound waves (a few millimetres in length). The sound waves were made plane by a wooden concave mirror and diffracted by the grating.

Examples

1. Give some examples of shadows in sound.

Explain clearly why sound shadows are generally not so well-marked at those of light. (*C. U. 1926*)

2. What is an echo? Why is succession of echoes sometimes observed?

You start walking away from a high wall clapping your hands once every second. How far must you go from the wall before you hear the echo of one clap simultaneously with the next clap?

(Speed or sound in air is 1120 ft. per sec.) (*C. U. 1928*)

3. Explain the production of echoes.

A gun is fired on the sea-shore in front of a line of cliffs. A man standing 800 feet away from the gun and equidistant from the cliffs notices that the echo takes twice as along to reach him as does the direct report. Find by calculation or graphically the distance of the gun from the cliffs. (Velocity of sound 1100 feet per second.) (*C. U. 1929*)

4. Explain the formation of an echo. Do you know of any use of the phenomena in obtaining scientific data about distant unseen objects? (*C. U. 1949*)

CHAPTER XI

RESONANCE AND RESPONSE

94. The principles of forced vibration and resonance have been treated in Chapter V. The agent that provides the periodic force is often known as the *driver* and the body which responds to the periodic force supplied is known as the *driven*. It has been deduced that when the period of the driver happens to be the same as that of the driven, the latter vibrates with maximum amplitude which in acoustics is observed by the large intensity of the sound produced by it.* The driven in this case fully responded to the driver.

FIG. 32

In this chapter we propose to deal with some phenomena based upon resonance, which have important bearing to certain acoustic determinations.

95. Air Resonator—The air resonator due to Wertheim, in its convenient form, consists of a glass tube T (fig. 32) connected to a reservoir R by a long india-rubber tube. The india-rubber tube, part of the reservoir and the tube T contain water. The tube T is fixed vertically alongside a scale and the reservoir can be moved up and down thus changing the length of the air column in the tube T. The apparatus was used by Wertheim in 1849 to deter-

* The energy of vibration is proportional to the square of the amplitude of vibration. See Art. 32 or the author's 'Text-Book of General Physics', Art. 40. The larger the amplitude, the greater is the energy of vibration and more intense is the sound produced.

mine the velocity of sound propagation in air and also by Blaikely in 1872 to find the correction at the open end of a tube.

If a vibrating tuning-fork is held near the open end of the glass tube, longitudinal waves from the tuning-fork travel down the tube and get reflected when they reach the surface of water in the tube. Since the water surface does not appreciably yield to the vibrations of the air, a pulse of compression or rarefaction is reflected, unchanged in type, from the surface of water (see Art. 88, *a*). The surface of water is thus the seat of a node of the stationary waves formed by the direct and reflected waves within the tube. But when the reflected waves reach the open end of the tube a compressed pulse is reflected as a rarefied one and *vice-versa* (see Art. 88, *b*) making the open end the seat of an *antinode* or *loop* of the stationary waves.

In reality, the open end of the tube is not strictly the antinode, for some sound energy escapes at each reflection from the open end and is radiated in the form of spherical waves from that end. The reflection thus occurs from a little distance beyond the open end and the seat of the antinode is a little distance away from the open end. The length of the air column in stationary vibration is a little greater than the length of the air column enclosed.

The distance of the antinode from the open end is called the *end correction** of the tube.

To carry out the experiment, the water reservoir is raised to reduce the length of the air column to a minimum.

*Helmholtz and Rayleigh have theoretically calculated the end correction for an infinitely flanged pipe. *Vide* Theory of Sound, Rayleigh, Vol. II, Arts. 307 and 314. But no theoretical calculation seems to have been made for an unflanged pipe.

A vibrating tuning-fork of known frequency is then held near the open end of the tube and the length of the air column is *gradually* increased by lowering the water reservoir until there is resonance produced between the tuning-fork and the vibrating air column. The resonance is detected by the sound of maximum intensity produced by the vibrating air column. The actual length of the air column from the level of water in the tube to the top of the tube is measured, this added to the end correction at the open end of the tube gives the distance between the node at the closed end and the antinode near the open end amounting to $\frac{\lambda}{4}$, where λ is the wave-length of longitudinal vibration of the air enclosed. If l_1 represents the actual length of the air column and x the correction, we have

$$\frac{\lambda}{4}=l_1+x \qquad \ldots \qquad \ldots \qquad \ldots \qquad (1)$$

The reservior is then lowered gradually to increase the length of the air column, with the vibrating tunning-fork held above it, until the air column is again in resonant vibration with the tuning-fork. This takes place when the length of the air column is such that there are one antinode and one node in the intermediate position.* The actual length l_2 of the air column added to the end correction amounts to $\frac{3\lambda}{4}$

$$\therefore \quad \frac{3\lambda}{4}=l_2+x \qquad \ldots \qquad \ldots \qquad \ldots \qquad (2)$$

Eliminating x from (1) and (2), we get

$$\frac{\lambda}{2}=l_2-l_1$$

$$\therefore \quad \lambda=2(l_2-l_1)$$

* See Art. 141.

Since resonance is occurring between the tuning-fork and the vibrating air column, their periods of vibration or frequencies are the same. Hence, if n represents the frequency of the tuning-fork, n is also the frequency of vibration of the air column and the velocity of longitudinal waves in the air enclosed is given by

$$V = n\lambda = 2n(l_2 - l_1)$$

The velocity thus determined corresponds to the temperature and humidity of the enclosed air. To reduce the velocity to dry air at $O^\circ C$, the usual corrections for temperature and moisture have to be made.* In making the moisture correction, the air enclosed within the tube may be taken to be saturated with water vapour due to the constant presence of a column of water below it, so that the value of f (Art. 46, c) may be taken as the saturation pressure of water vapour at the temperature of the enclosed air.†

96. End Correction—The above method was employed by Blaikley in 1879 to determine the correction at the open end of a tube. Determining l_1 and l_2 in the same way and eliminating λ from equations (1) and (2) of Art. 95,

$$3(l_1 + x) = l_2 + x$$

$$\therefore\ 2x = l_2 - 3l_1$$

$$\text{or } x = \tfrac{1}{2}(l_2 - 3l_1)$$

$$= \tfrac{1}{2}(l_2 - l_1) - l_1$$

The value of x lies near about $\cdot 6r$, where r is the radius of the tube.

*See Art. 46.

† For Wertheim's method of finding the velocity of sound in liquids, see Art. 139.

7. **Kundt's Tube**—A tube method of finding the velocity of longitudinal waves in a gas or in solid, when one of the two is known, was devised by Kundt in 1866. The apparatus consists of a long wide glass tube closed at one end by a tightly fitted piston A (fig. 33). The interior of tube known as the *wave tube* is well dried and strewed with dry lycopodium seed or cork dust. The tube is placed horizontally on suitable supports and its other end is provided with a loose piston B at the end of a metal or glass rod C, called the *sounding rod*. The rod is firmly clamped exactly at the middle point of it and placed so that the axis of the rod and that of the tube are in the same line, the piston B being within the tube.

FIG. 33

To perform the experiment, strike the rod with a rosined leather if the rod be metal or with a moist cloth if of glass, and at the same time adjust the position of the piston A, by pushing it in or pulling it out, until a clear loud note rings out. Longitudinal vibrations thus produced in the rod set the air within the glass tube into resonant vibration. The longitudinal waves starting from the sounding rod and travelling within the enclosed air get reflected from the surface of the piston A, and due to the direct and reflected waves, stationary waves, having fixed nodes and loops, are set up within the tube. The fine lycopodium seeds fly away from the loops, the places of maximum displacement of air particles to nodes, the places of minimum displacements, and collect in heaps at the nodes. The mean

distance between two consecutive nodes within the glass tube is measured. If this amounts to l, the wave-length of the longitudinal vibration of air is $2l$. The sounding rod in its simplest mode of vibration has a node at the middle where it is clamped and two loops at the two ends, so that if l' be the length of the rod, the wave-length of longitudinal wave within the rod is $2l'$. Since the rod and the tube are in resonant vibration, they have the same frequency of vibration. Representing the velocity of longitudinal waves in air by V and that within the rod by V' we have

$$\frac{V}{V'}=\frac{n\times 2l}{n\times 2l'}=\frac{l}{l'}$$

where n is the common frequency.

The above relation provides a method of finding V or V' if one is known.

The air within the wave tube may be replaced by any other gas and the velocity of sound in the gas found out.

Kundt's tube provides a ready method for finding the velocity of sound in a gas under controlled conditions of temperature and pressure. Kundt showed by this method that the velocity of the sound in a gas was independent of pressure and proportional to the square root of absolute temperature of the gas.*

For the success of the experiment several practical details need careful attention, *viz.* (1) The rod must be clamped exactly at its middle point. (2) The position of the piston A must be adjusted properly for the formation of resonance within the tube. (3) Finally, the dust figures will not be produced, if the dust or tube is wet or too much dust is taken.

*See Art. 46.

Kundt later used a double from of the apparatus, in which the same rod was used to excite resonant vibrations in *two* wave-tubes, one at each end. The sounding-rod was excited by rubbing near the middle. This method is suitable for *comparing* the velocities of sound in two different gases.

The two wave-tubes contained the two different gases which were excited to resonant vibrations. If d_1 and d_2 are the respective nodal separations in the tubes and V_1 and V_2 are the corresponding velocities of sound in the gases contained, $V_1/V_2 = d_1/d_2$.

A number of modified forms of Kundt's tube have been developed, but the simple apparatus shown in fig. 33 is however, very effective in demonstrating the state of vibration of the resonating gas column contained in the tubes.*

98. Besides finding the velocity of sound in a gas or in a solid, the experiment serves to find the ratio of the specific heats of a gas at constant pressure and at constant volume from the relation

$$V = \sqrt{\frac{\gamma P}{\rho}}$$

Kundt and Warburg determined the ratio of specific heats of mercury vapour by the method in 1876. Rayleigl and Ramsay determined γ for argon and helium by this method in 1895 and established their monatonic nature. †

Recently, Andrade and Lewer ‡ (1929 and 1930) have obtained much improved effects by employing a telephone

* Vide 'A Text-Book of Sound', Wood. p. 179.

† See author's text-book of General Physics, art, 166.

‡ See 'Nature', 9th Nov. 1929 and Journal of Sci. Instruments Feb, 1930.

diaphragm to excite the wave-tube in resonance. The loud speaker is supplied with an alternating current, the frequency of which is adjusted until the gas in the wave-tube is thrown into resonant vibration.

Kundt and Lehmann applied the tube method in finding the velocity of sound in liquids. Fine iron filings or powdered sand were strewn along the tube. The tube was excited to resonance in usual way when the iron filings collected at the nodes.

Green has recently in 1923 measured the velocity of sound in liquids contained in a tube, one end of which is closed by a steel diaphragm. The diaphragm is excited to any desired frequency by a small electromagnet supplied with alternating current. The powdered sand within the tube collects at the nodes from which the velocity is calculated.

99. Electrically Maintained Tuning-forks—The electrically maintained tuning-fork is a convenient example where the vibrations of the tuning-fork are sustained by the expenditure of electrical energy. In one form of it, due to Helmholtz, an electromagnet is fed by a current from a battery. The electromagnet attracts the prongs of the tuning-fork causing them to separate. Due to the outward otion of the prongs, interruption is made in the electrical circuit by a platinum wire dipping into a cup of mercury.

Another form of the electrically maintained tuning-fork, which is now in common use, is shown diagrammatically in fig. 34. The tuning-fork is clamped firmly on a strong support. To one prong of it, is attached a light platinum pointer *w* touching an adjustable metal screw *S*. Electric

circuit is established through this prong, the pointer, adjustable screw and finally through an electromagnet *M* placed

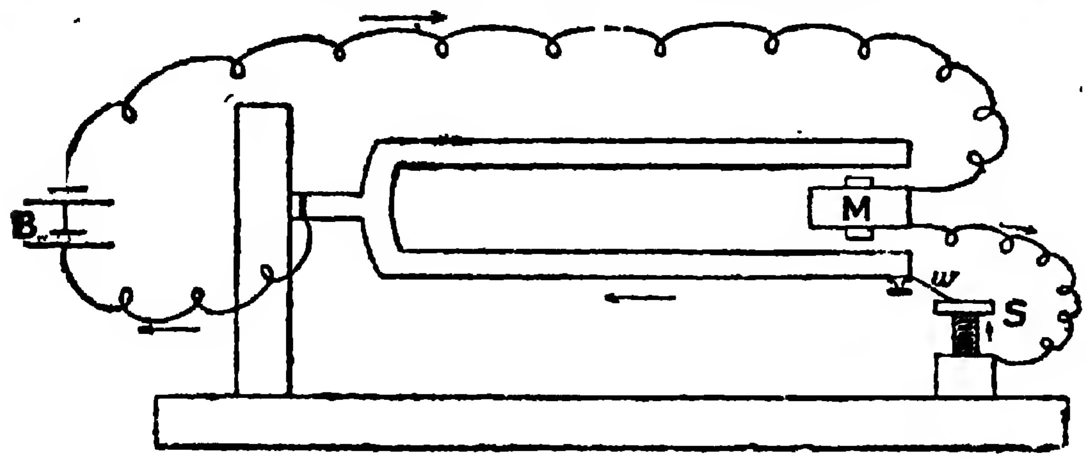

FIG. 34

between the prongs of the tuning-fork. As the electromagnet is excited, it attracts the prongs together causing a break in the electric circuit. The prongs then move apart due to elastic forces and the electric circuit is re-established by the platinum pointer coming in contact with the adjustable screw. The alternate interruptions, produced in this way, maintain the vibrations of the tuning-fork. This method of maintaining the vibrations of a fork ceases to be efficient when the frequency of the fork exceeds about 100 cycles per second.

Eccles* has described a method of driving a tuning-fork by means of a thermionic valve which is particularly applicable to forks of higher frequencies.

100. Maintenance of Vibrations by Heat.—As in the electrically maintained tuning-fork, mechanical vibrations are maintained at the expenditure of electrical energy,

* See 'Modern Acoustics' by Davis and Proc. Phys. Soc., 31, 269, 1920,

similarly, in many cases, such as in the hydrogen singing flame, Trevelyan rocker, etc., vibrations are maintained at the expenditure of heat energy. Lord Rayleigh successfully explained the phenomenon of the singing flame in 1878.*

In 1831 Trevelyan accidentally discovered that a hot iron fork placed on a cold block of lead gave rise to a musical note. To show the effect in a convenient manner a prism of brass or copper of triangular section is taken. One of its edges is removed and a groove is made in the small flat thus produced. The end of the prism is carried by a brass rod terminating in a knob. This is known as the rocker.

A block of lead preferably with a rounded top is taken and its top surface is cleansed with steel scraper. The rocker is then heated over a Bunsen flame to some temperature below the melting point of lead. It is then quickly placed with its groove downwards on the lead block

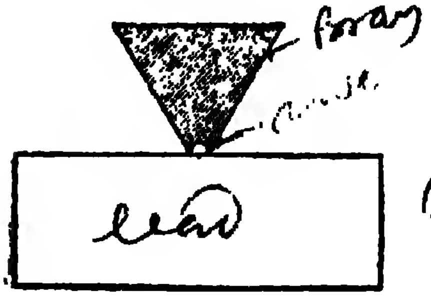

FIG. 35

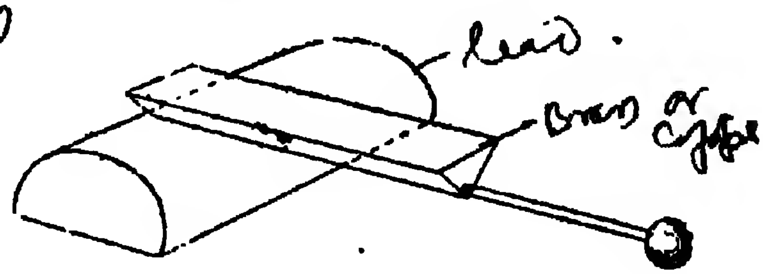

FIG. 36

(figs. 35, 36). At first the rocker may be too hot to work properly but after a little trial when matters are rightly adjusted the rocker will rock rapidly and emit a musical note.

* For this the student is referred to 'The Theory of Sound,' Rayleigh Vol. II. Art. 322 or Poynting and Thomson's Sound, p. 185.

The explanation of the phenomenon is given in this way.—The rocker is a good conductor of heat. The metallic faces of the groove which should be very clean rapidly communicate heat to the lead as brass is a good conductor to heat. But as lead is not a very good conductor of heat, the heat does not diffuse rapidly into the lead but remains near the point of contact causing local expansion. When the rocker is placed with one of its narrow faces in contact with the lead a slight hump is raised on the lead due to local expansion. This tilts the rocker to its other face where the operations are repeated. The rocker then returns to its original position and the vibrations of the rocker continue until the lead becomes uniformly heated by transmission of heat from the rocker.

The experiment is also found to succeed if the rocker is made of cold lead and placed on a hot copper block.

Example

1. Describe a method by which the velocity of sound in a gas may be determined and compared with those in other gases.

The velocity of sound in air at $0^\circ C$, is 332 metres per second. Find the shortest length of a tube, open at both ends, that will be thrown into resonant vibration by a fork whose frequency is 256, when the temperature of the air is $51^\circ C$. *(C. U. 1915)*

2. Give a brief account of Kundt's experiment. If the length of the rod is 1 metre, the density of the material 8 grms. per c. c. and its Young's Modulus $7{\cdot}2 \times 10^8$ grams. per sq. cm., find the distance between the heaps in the Kundt's tube when it is filled with CO_2 at $25^\circ C$.

[Velocity of sound in CO_2 at $t^\circ C$

$$= 260 + 0{\cdot}478t \left(\frac{\text{metres}}{\text{sec.}}\right)]$$ *(C. U. 1917)*

3. Explain briefly the resonance column method for finding the velocity of sound in air.

The velocity of sound in air at $30°C$ and saturated with aqueous vapour is found to be 350 metres per second. Calculate the velocity [illegible]ry air at N. T. P.

Mass of 1 c. c. air at N. T. P. = 0·001293 gramme.

Barometric height (corrected) = 760 mm.

Maximum vapour pressure at $30°C$ = 31·5 mm. *(C. U. 1920)*

4. Briefly explain the theory and the method of finding the velocity of sound in air by the resonance column.

Calculate the velocity of sound in dry air at $0°C$, and 760 mm. pressure from the following data, supposing the density of moist air inside the tube to be 0·00120 grm.

Length of the column of air for the

1st maximum = 33 cm.

Length of the 2nd maximum = 101·6 cm.

Temperature of air inside the tube = 30 C.

Barometric height (corrected) = 760 mm.

Frequency of the fork = 256. *(C. U. 1922)*

5. Describe in detail an experimental arrangement for the determination of the velocity of sound is gases at different temperatures and pressures.

How does the velocity vary with pressure? Give reasons for your answer. *(C. U. 1935)*

6. Describe experiments illustrating the maintenance of vibrations by heat and electricity. Explain in a general way the modes of maintenance. *(C. U. 1943)*

7. Describe experiments illustrating the maintenance of vibrations by heat and electricity. *(C. U. 1952)*

CHAPTER XII

MUSICAL SOUND

101. The Three Characteristics of Musical Sound—A musical sound possesses three definite characteristics and our sound sensations differ from one another in these three respects only, *viz. Intensity, Pitch and Quality.*

102. Intensity—The intensity of a sound, whether musical or unmusical, is the other name for its *loudness.* If the source of a sound vibrates with greater vigour. *i. e.* with an increased amplitude of vibration, the sound produced by it will be louder or more intense. It has already been said (see Arts. 32 and 94, foot-note) that the intensity of a sound is directly proportional to the square of its amplitude and frequency of vibration. The intensity of sound emitted by a source falls off with distance from the source obeying the *inverse square law* as the intensity of light. To establish this, consider a sound source at O (fig. 37) sending out Q units of sound energy per second equally in all directions. Imagine two spherical surfaces with centre O and passing through P and R at distances r_1 and r_2 respectively from O. Since there is no accumulation of sound energy anywhere round O, it is evident that the same sound energy Q is passing out per second through either sphere. Therefore the intensity of sound at P, which

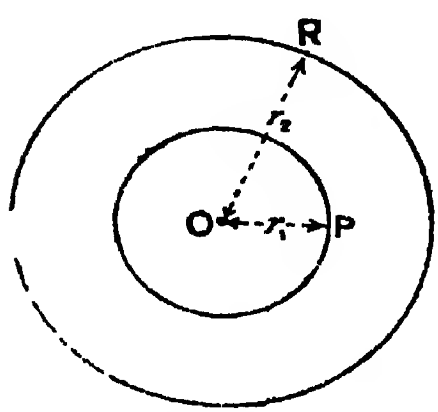

FIG. 37

is measured by the amount of sound energy falling on unit area of the sphere at P, is $Q/4\pi r_1^2$. Similarly, the intensity of sound at R is $Q/4\pi r_2^2$

$$\therefore \frac{\text{intensity at } P}{\text{intensity at } R}=\frac{Q/4\pi r_1^2}{Q/4\pi r_2^2}=\frac{r_2^2}{r_1^2}$$

or the intensity $\propto \frac{1}{(\text{distance})^2}$.

Lord Rayleigh* has made an absolute measurement of the intensity of the faintest sound that is audible. Topler and Boltzmann † devised an optical method for the same.

103. Pitch—The pitch is a physical cause which distinguishes a treble note from a bass note of the same intensity on the same musical instrument. The former is said to possess a higher pitch than the latter. The pitch is a fundamental characteristic of a musical sound. A noise has no definite pitch. The pitch of the note emitted by a source depends upon its frequency of vibration. The higher the frequency of vibration of a source, the more shrill is the sound emitted by it and the sound rises in pitch. As the pitch of a note is directly proportional to its frequency, it is customary in science to express the pitch of a note by its frequency.

The measurement of the pitch or frequency of a note can be made in a number of ways some of which are given below

104. Chronograph—The chronograph due to Professor A. M Meyer is a helpful device for the *absolute* measurement

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*See "Theory of Sound', Lord Rayleigh, Vol. II, p. 484.

† See 'Catstick's Sound,' Art. 245, or 'Modern Acoustics', Davis, p. 110.

of the frequency of a tuning-fork. A cylindrical drum D (fig. 38) can be rotated about an axle S which is provided with a handle. On the axle, screw-thread is cut, so that when the handle is turned, the drum rotates and at the same time advances along the axle. A piece of smoked paper is wrapped round the drum. The tuning-fork is driven electrically and to one of its prongs a style is attached.

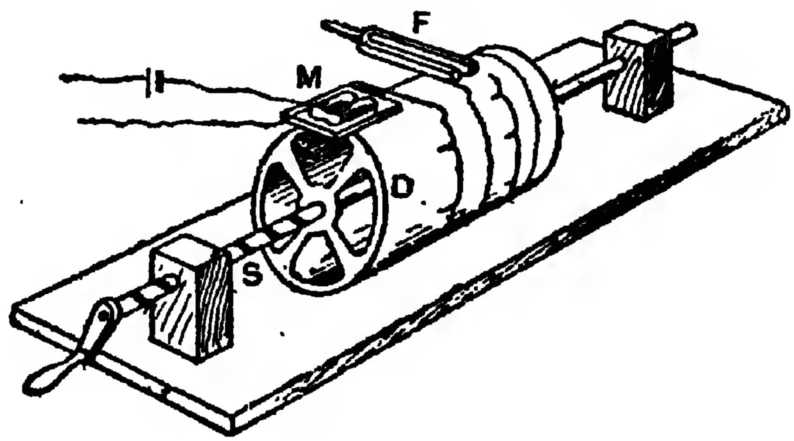

FIG 38

The style touches the drum lightly and leaves a wavy curve on the smoked paper without overlapping as the drum rotates and advances along the axle.

In order to measure intervals of time a small electromagnet is provided with a style at its armature. The electromagnet is excited by a battery only when a clock pendulum beating half seconds passes through its lowest position. The pendulum in passing through its lowest position touches the surface of mercury contained in a cup and closes the circuit. When the electromagnet is not excited, the style attached to its armature, draws a line alongside that due to the fork, but it gives a little kick when the electric contact is made at intervals of half a second. The number of vibrations of the fork between any

half-second marks upon the smoked paper is counted and the frequency of the fork is calculated. This arrangement is known as chronograph.

105. Stroboscopic Method—If a body is in periodic motion and if it be intermittently illuminated, such that the period of intermittence coincides with the period of the body, the body will appear motionless. Devices of this kind are known as *stroboscopes*. The attention of a number of scientists was drawn in the early part of the nineteenth century but an axact measurement with a stroboscope seems to have been first made by Topler in 1866.

A form of stroboscope suitable for measuring the frequency of a tuning-fork is shown in fig. 39. A disc D,

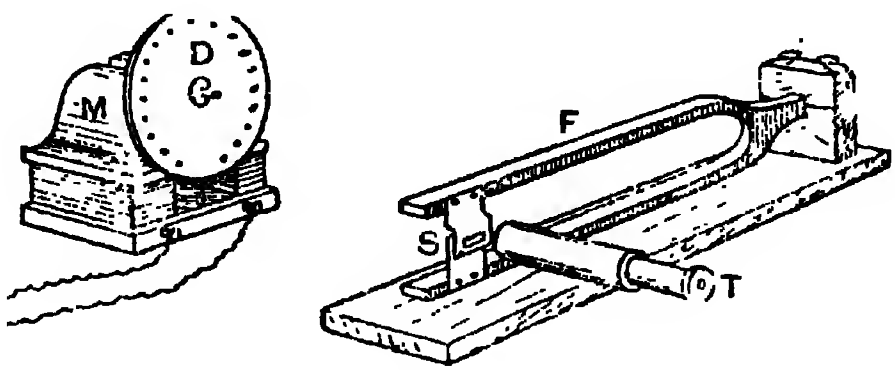

FIG 39

known as the stroboscopic disc, has a circle of equally spaced dots fixed near its periphery. The disc is fitted to the armature of an electric motor M and is focussed by a telescope T. Before the objective of the telescope is fixed the tuning-fork which is driven electrically. Two light screens are attached to the prongs of the tuning-fork. Each screen has a slot cut in it, such that when the prongs are at rest, the dots on the disc D can be seen by the telescope through the slots.

The fork is now set to vibration and the disc is driven and its speed of rotation is *gradually* increased from a low value until the dots appear to be stationary when seen through the telescope. This will evidently be the case when the time between the arrivals of two successive dots in front of the cross-wire of the telescope is the same which elapses between two successive coincidences of the two slots in the screens. Again, the time between two successive coincidences of the slots is half the period of vibration of the tuning-fork. By measuring speed of revolution of the disc with a speed counter and counting the number of dots on the disc, the time between two successive dots occupying the same position before the telescope is measured from which the frequency of the tuning-fork is found out.

If the disc is rotated too slowly, each dot will be a little behind the position occupied by the one in front when last seen, and the dots will appear to move slightly backwards. If rotated too rapidly, the dots will appear to move slightly forwards. It will, however, be noticed that if the speed of rotation of the disc be increased to double or to three times its previous value, the dots will again appear to be stationary. Therefore, in determining the frequency, the speed of rotation must be very gradually increased from a low value until the dots appear to be stationary.

106. Comparison Method—The pitch of a note is determined by comparing it with another produced artificially by suitable device and adjusting the pitch of the latter so as to bring it in unison with the note. The ear is particularly adapted to distinguish between the two notes having different pitches. When unison is obtained,

the pitch of the note in question is evidently the same as that of the note produced. The following apparatuses are suitable for the purpose.

(1) *Savart's Toothed Wheel.* This method due to Savart consists of a brass wheel having teeth cut on its edge (fig. 40). The wheel is mounted at its centre on an electric motor and is rotated at a uniform speed which can be varied. A card or plate, held against the teeth, gives out a note the pitch of which can be varied by varying its speed of rotation. The speed of rotation is adjusted until the note produced is in unison with the note in question. The speed of rotation of the wheel is found out by a speed counter.

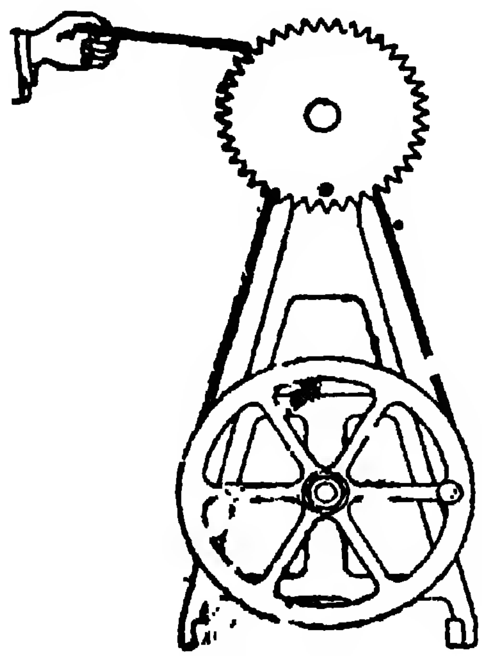

FIG. 40

If the wheel is rotated m times per second and has n teeth, the pitch of the note produced or that of the note under examination is $m.n.$ For a wide range of variation, there are several brass wheels having different numbers of teeth, mounted on the same spindle.

(2) *The Siren*—(*a*) The *disc siren* due to Seebeck consists of a disc having several rows of equally spaced concentric holes (fig. 41), the different rows having different numbers of holes. The disc is mounted on an electric motor and rotated at a uniform speed which can be varied. Air from a foot blower is passed through a fine nozzle held before a hole on the disc. If the speed of rotation of the

disc is slow, puffs of air pass through a hole when it comes before the nozzle, but the air is obstructed when a space comes before it. This periodic obstruction of the air gives out a note when the speed of rotation is sufficiently large. By adjusting the speed of the disc and by changing the row of holes, the note produced may be adjusted to be in unison with the note to be examined. If the particular row has m holes and if the disc is rotated n times per second, which is measured by a speed counter, the pitch of the note produced or that of the note in question is $m. n.$

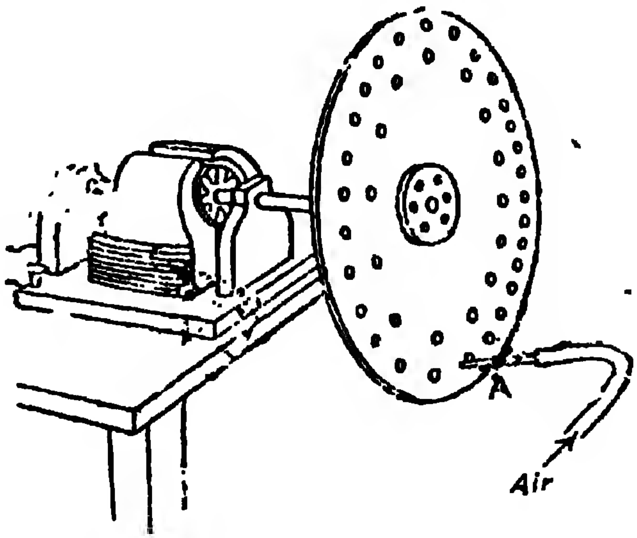

FIG 41

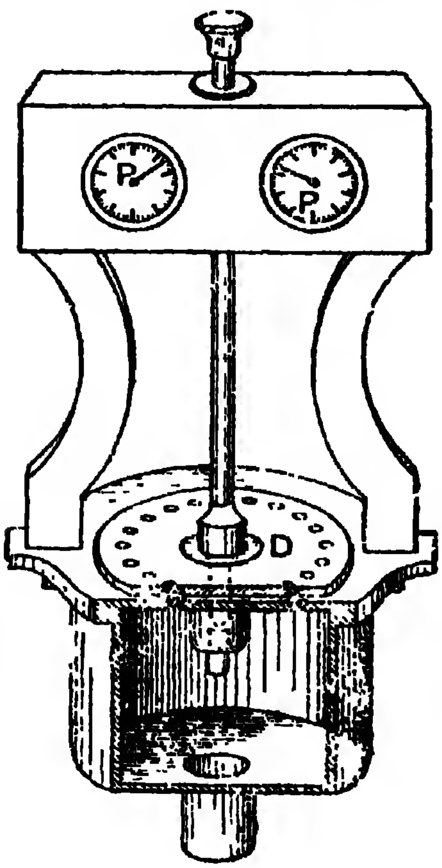

FIG. 42

(*b*) Another form of siren due to Cagniard de la Tour, in which a single jet of air is replaced by a ring of jets, is a convenient one. It consists of a vertical spindle carrying a circular disc pierced with a row of equally spaced holes on the circumference of a concentric circle (fig. 42). The disc can be rotated about the vertical spindle and has below it another disc provided with a similar set of holes. The latter disc is

kept fixed on the top of a cylindrical windchest. The two rows of holes in the two discs have obliquities turned oppositely so that the pressure of air, as it comes out through the holes of the fixed disc, spins the former disc. By adjusting the pressure of air, the disc can be rotated at any desired speed. The alternate obstruction of the air gives out a note the pitch of which can be adjusted to be in unison with that of the note in question. The number of rotations per second of the disc is counted from a counting arrangement at the top of the spindle. If the disc rotates m times per second and if there are n holes in the row, the pitch of the note produced by it or of the note in question is $m. n.$

In this form of the siren, the intensity as well as the pitch rises with higher speed of rotation. The wave form of the note of the siren indicates that along with the fundamental a number of harmonics is also present. Milne and Fowler* have recently devised a special form of siren of Seebeck type giving pure tones.

(3) *Sonometer*—A simple form of sonometer has been described in art. 69. In order to determine the frequency of a tuning-fork, the string is stretched under a tension ; the positions of the movable bridges are varied and the length of the vibrating segment between them adjusted, until the string is set to resonant vibration when the vibrating tuning-fork is held on its sound board. If l be the length of the vibrating segment, T the tension of the string in absolute units and m the linear density of the string, the

*Proc' Roy, Soc. 98, p. 414, 1921.

frequency of the note emitted by the string or that of the tuning-fork is given by

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

A more convenient form of sonometer, known as the 'vertical sonometer,' referred to in Art. 70, consists in suspending it vertically from a strong support. As the load is applied to the vertically suspended string, the friction of the pulley has hardly any effect to lossen the stretching force.

A form of sonometer which can be excited by an electro-magnet has been designed by D. W. Dye in 1924. A small alternating current passes through the wire made of phosphor bronze of a vertical sonometer. A portion of the wire lies between the poles of a permanent magnet which can slide along the length of the wire. The stem of the vibrating tuning-fork is pressed against the sound board, the length of the vibrating segment is varied and at the same time the permanent magnet is slided a little along the wire, until the wire is set to resonant vibration. The wire can be set to vibration in any number of loops provided the magnet does not coincide with a node.

107. Indirect Method—The frequency of a tuning-fork is determined by the air resonator described in Art. 95. The fork is set to vibration and held before an air resonator the length of which is adjusted to resonance. For the fundamental vibration of the air column, the velocity of sound in the air within the resonator is evidently given by

$$V = 4n(l + \cdot 6r) \qquad \text{(See Art. 96)}$$

where n is the frequency of the fork, l the length of the air column and r the radius of the tube. The velocity V can

be determined previously with a tuning-fork of known frequency, as described in Art. 95, and n calculated from the above relation.

108. Quality—Besides intensity and pitch, there is a third feature of musical sound which enables our ear to distinguish between two notes of the same intensity and pitch but arising from two different musical instruments. The discovery of this feature of musical note is due to Helmholtz and known as *quality*, *character* or *timbre*. If a musical note of an assigned pitch is produced by a musical instrument, the note is never of one single frequency, *i. e.* not a tone but it is a mixture of a prominent fundamental having the assigned frequency and some overtones of relatively small intensity which are generally the harmonics of the fundamental.* An almost pure tone is produced by the tuning-fork, a fact which makes it so important in acoustical determinations.

Helmholtz's conclusion on quality is that the quality of a musical note depends only on *number*, *order* and *relative strength* of its harmonic constituents but *not on their difference of phase.*†

It is the difference in harmonics produced by different musical instruments along with the same fundamental which distinguishes between the same two notes in them.

The presence of harmonics affects the form of the displacement curve of the wave. The displacement diagram

* The presence of overtones which are not harmonics of the fundamental makes the note unmusical, see Art. 78, the transverse vibration of rods.

† See Art. 111.

of a simple sound is simple harmonic wave curve. When harmonics are present, the form of the curve becomes complex. The complexity increases with the increase in number of the harmonics present. The adjoining figure (fig. 43) is drawn with a fundamental and its first harmonic.

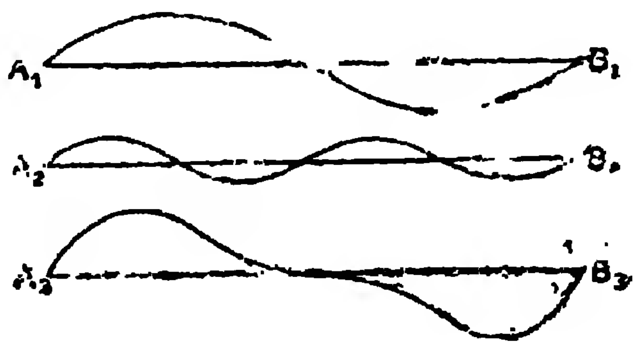

FIG. 43

A_1B_1 shows the fundamental, A_2B_2 the first harmonic of the fundamental and A_3B_3 the resultant of the two.

Our ear is cabable of resolving the complex wave. The overtones present determine the quality of the note and enable the listener to ascertain the source of the note.*

109. Fourier's Theorem—The combination of a number of collinear S. H. M.'s of commensurate periods is a periodic motion.† The French mathematician, Fourier, enunciated a converse theorem of this in 1822, which can be briefly stated as 'any single valued periodic function, whatever, can be expressed as a summation of simple harmonic terms having frequencies which are multiples of that of the given function'.

The theorem has two limitations regarding the nature of the complex vibration, *viz.* (1) that the displacement

*See Art. 149.

†See Author's Text-Book of General Physics', Art. 52.

must be single valued and continuous and (2) that the displacement must always have a finite value. It is obvious that these limitations do not concern us in dealing with sound waves ; for, a particle cannot have two displacements at the same instant of time nor can its displacement be infinite.

Fourier's theorem has ample application in different branches of heat and acoustics. In acoustics, it is applied for the analysis of musical notes. A complex musical note may be graphically represented as a periodic function. Fourier's theorem states that it is made up of a number of pure tones. If the musical note is of frequency n, it is a mixture of a fundamental of frequency n and of some of the tones having frequencies $2n$, $3n$, etc., the intensity of the fundamental being large compared to that of any of the higher tones.

110. Fourier's theorem is susceptible of many forms of expression. For Fourier's method of treating the theorem, mathematical text-books on the subject must be consulted. The theorem can be put down in the following form.

Let y be a periodic function of x such that y has one and the sames value as x changes by λ, then

$$y = A_0 + A_1 \cos\frac{2\pi x}{\lambda} + A_2 \cos\frac{4\pi x}{\lambda} + \dots\dots\dots$$

$$+ B_1 \sin\frac{2\pi x}{\lambda} + B_2 \sin\frac{4\pi x}{\lambda} + \dots\dots\dots$$

To evaluate the coefficient A_0, multiply both sides by dx and integrate between the limits $x = O$ to $x = \lambda$. Evidently all the terms vanish except the first one, and

$$\int_0^\lambda y\,dx = A_0 \int_0^\lambda dx = A_0\lambda$$

$$\therefore \quad A_0 = \frac{1}{\lambda}\int_0^\lambda y\,dx \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

To evaluate A_n, multiply both sides by $2\cos\frac{2\pi nx}{\lambda}dx$ and integrate between the same limits

$$\therefore \quad 2\int_0^\lambda y\cos\frac{2\pi nx}{\lambda}dx = A_n\int_0^\lambda 2\cos^2\frac{2\pi nx}{\lambda}dx$$

(the other terms on the right side vanish)

$$= A_n\int_0^\lambda\left(1+\cos\frac{4\pi nx}{\lambda}\right)dx$$

$$= A_n\int_0^\lambda dx + A_n\int_0^\lambda \cos\frac{4\pi nx}{\lambda}dx = A_n\int_0^\lambda dx$$

$$= A_n\lambda$$

(as the other integral vanishes)

$$\text{whence} \quad A_n = \frac{2}{\lambda}\int_0^\lambda y\cos\frac{2\pi nx}{\lambda}dx \quad \ldots \quad \ldots \quad (2)$$

$$\text{Similarly,} \quad B_n = \frac{2}{\lambda}\int_0^\lambda y\sin\frac{2\pi nx}{\lambda}dx \quad \ldots \quad \ldots \quad (3)$$

Relation (1) gives the value of the coefficient A_0. The coefficients A_1, A_2, A_3, etc., can be found out from relation (2) replacing n by 1, 2, 3, etc., successively. Similarly, relation (3) admits of the evaluation of B_1, B_2, B_3. etc.

The terms of the right-hand side excepting the first are simple harmonic having frequencies as 1 : 2 : 3, etc. These terms with the values of the coefficients determined from (2) and (3) give the periodic function y. The first term is an absolute one independent of x.

111. Quality and Phase—Helmholtz's conclusion on the quality of a musical note (Art. 108) states that the factors governing the quality of a musical note are the number, order and strength of the harmonics present with the fundamental. Any one or more of these affects the form of the wave. The form of the wave may, however, be altered by a mere difference in the phases of the components of the note although their number, order and strength are maintained the same.

In figures 43 and 44, A_3B_3 represents the resultant of the two components $A_1 B_1$ and $A_2 B_2$ drawn respectively

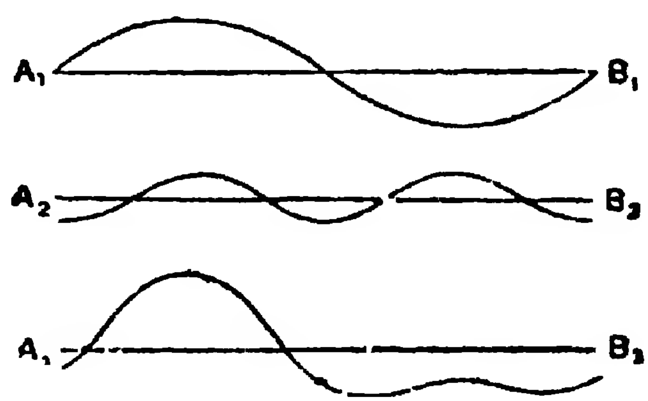

FIG. 44

above them. It will be found that in both the figures A_1B_1 and A_2B_2 have the same amplitude and are of the same order, *viz.* A_2B_2 is the octave of A_1B_1, but in the two figures A_1B_1 and A_2B_2 appear in different phases. The wave-form A_3B_3 of the resultant curve is different in the two figures. In order to test whether a change of wave-form due to a mere change in phase of the component tones produces a change in quality, experiments were made by Helmholtz*. A set of tuning-forks with corresponding

* See 'Sensations of Tone' Helmholtz. p. 108, etc.

resonators of special type was constructed. When any combination of these forks was made to sound and the phases were varied, while the intensities were kept practically constant, the variations in phase could not be observed to produce any change in quality; while the quality was quite altered by changes in the number, order and strength of the constituents. König in 1881 constructed a special siren for the purpose and came to the conclusion that a mere change in phase of the constituents of a musical note did not affect its quality. With the development of polyphase alternating currents, Lloyd and Agnew, more recently (1909) were able to reverse the phase of one component of a vibration on a diaphragm transmitter. They found no change in quality of the note heard.

112. Analysis of Notes—With experience it is possible to analyse a note into its harmonics to some extent by the unaided ear. Such an analysis is untrustworthy, because of our habit of disregarding the harmonics of a note and concentrating our attention on the fundamental. A type of analyser devised by Helmholtz is specially suitable for sound analysis. The analyser is based upon the principle of resonance and is known as Helmholtz *resonator.*

113. Helmholtz Resonator.—It consists of a volume of air enclosed in a spherical (fig. 45) or cylindrical (fig. 46).

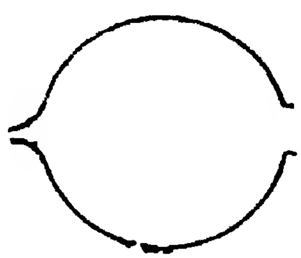

FIG. 45

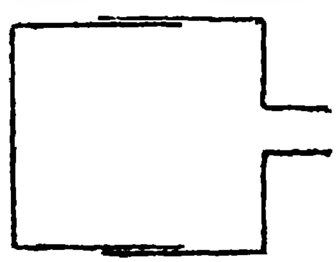

FIG. 46

vessel provided with a 'neck'. When a vibrating source of

sound is held near the neck of the resonator, the chief vibratory system of the resonator is the air within the neck of the resonator, the air within the body of it simply provides a spring action on the vibrating system. The air cavity of the resonator being almost completely enclosed, only a small proportion of its energy is radiated into the medium ; the result is that the damping is very small and the tuning is very sharp. For this, it is particularly adapted to detect sound waves of definite frequency. Two forms of the resonators are used,—in one, fig. 45, the length of the neck is negligible compared to its diameter and in the other form, fig. 46, the length of the neck is comparable to or larger than its diameter. The exact form of the cavity is immaterial,—it may be spherical or cylindrical in shape, provided its volume is considerably large compared to that of the neck.

It can be shown that when such a resonator vibrates the frequency n of the fundamental tone is given by

$$n=\frac{V}{2\pi}\sqrt{\frac{A}{LS}}$$

where V=the velocity of sound in the gas within the resonator,

A=cross section of the neck,

L=length of the neck,

and S=volume of the cavity.

The ratio A/L is called the *conductivity of the neck* from electrical analogy.

To determine the components of a complex musical note, Helmholtz used a graduated series of resonators of

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* See 'A Text-Book of ound', Barton. p. 821.

various volumes of cavity and areas of the neck. A large number of resonators is however necessary to cover a moderate range of frequency. It is therefore customary to use resonators of continuously variable volume which is achieved by means of a sliding piston forming the closed end of a cylindrical cavity. With change of volume of the cavity of the resonator its frequency of vibration changes. If a complex note, falling upon a particular resonator, sets it in resonant vibration, the resonator 'speaks' and evidently a tone or simple sound of the frequency of the resonator is present in the complex note. In this way the various components of the complex note can be found.

The resonance in a particular resonator can be detected by connecting a small opening, called '*pip*', at the base of the resonator to the ear, either directly or through a tube. It is better detected by connecting to the König's manometric flame described below.

114. Manometric Flame.—To study the vibrations in

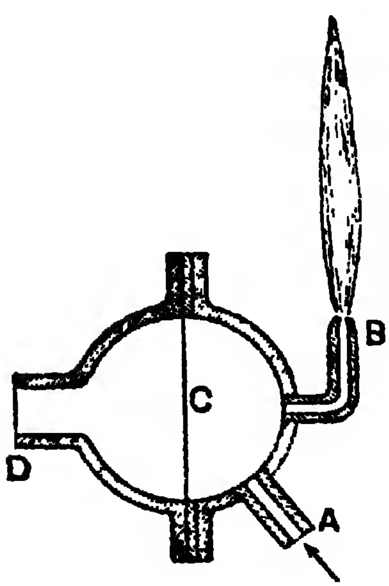

FIG. 47

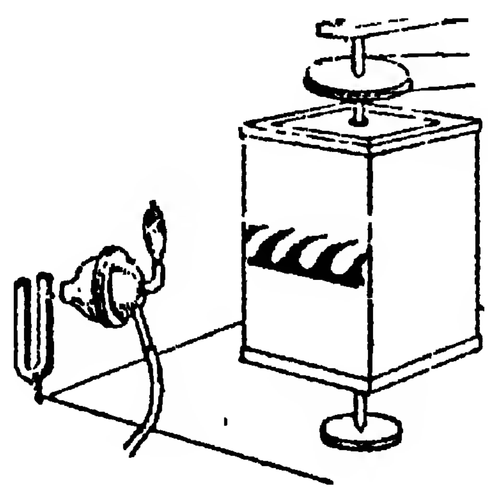

FIG. 48

pipes König devised a flame in 1872, showing the variations

of pressure ; the flame is known as the manometric flame. A small hollow chamber is divided into two parts by a thin sheet of india-rubber C (fig. 47). One of these parts is provided with an inlet A for coal gas which then passes to a pinhole burner B. The other part of the chamber is provided with an opening at D which is connected to a Helmholtz resonator or exposed to any other vibrating source, and the pinhole burner is lighted. The variations of pressure on the membrane, due to the vibrations of the resonator, cause the membrane to vibrate and check or aid the flow of gas to the burner. The flame thus jumps up and down with a frequency which is the same as that of the membrane. The movements of the flame are, however, too rapid to be seen separately due to persistence of retinal impressions. To show these, Wheatstone applied a cubical box with a plane mirror forming each of its vertical sides (fig. 48). The box is rotated near the flame about a vertical axis. The reflection consists of a band of light having toothed appearance.

In a modern device, due to Richardson in 1928, the flame and the inlet for gas are dispensed with, and to the surface of the rubber membrane R, a small mirror M is attached with rubber cement midway between the centre and the edge of the membrane (fig. 49). A narrow beam of light is reflected from the mirror to a distant scale. The vibrations of the membrane give a slight angular motion to the mirror and the spot of light is drawn out into a band.

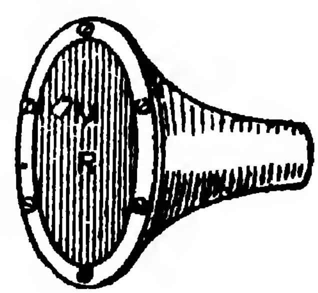

FIG. 49

The device Hot-wire microphone,* discovered by Tucker

* For this, the reader is referred to Richardson's Sound, p. 218.

and Paris in 1921, is a more suitable arrangement for detecting the components of a complex note.

Examples

1. What is pitch in the case of sound?
How is it accurately determined? *(C. U. 1911)*

2. Define wave-length and pitch of the musical note. How are they related? Explain a method of determining each.

When two notes of nealry equal pitch are sounded together, what effect is produced in the ear and in the air outside? *(C. U. 1914)*

3. On what do the loudness, the pitch and quality of a musical sound depend? Explain briefly how these may be graphically represented. How would you propose to determine the pitch of a note experimentally? *(C. U. 1915)*

4. Explain the method of finding the absolute frequency of a tuning-fork by a sonometer. What are the different sources of error, and how may these errors be minimized? *(C. U. 1922)*

5. Clearly distinguish between overtones and harmonics. Give a few instances of overtones that are not harmonics.

How can you detect the presence of harmonics in a musical note, and what are their functions? *(C. U. 1923)*

6. How has it been concluded that a sounding body, *e.g.* (*a*) a tuning fork, (*b*) the air column in an organ pipe, is in vibratory motion? To what physical characteristics do the loudness and pitch of a musical note correspond? *(C. U. 1925)*

7. Explain clearly the chief characteristics of a musical sound? Show that the intensity of sound emitted by a vibrating body is proportional to the square of the amplitude and the square of the frequency of vibration. *(C. U. 1941)*

CHAPTER XIII

INTERFERENCE, BEATS AND COMBINATIONAL TONES *

115. Interference—The term interference, is used in acoustics and optics to denote the phenomena of alternate reinforcement and annihilation of sound and light which occur under suitable conditions when radiations from two sound or light sources mingle. It is shown in treatises on physical optics that if there are two small sources of light emitting light waves of the *same* frequency and amplitude, they produce maximum illumination at a place on a screen where the waves of the two sources reach at the same phase ; on the other hand, the waves produce darkness at a place where they reach at opposite phases. With sound waves a similar state of affairs takes place. This interference phenomenon, as has already been referred to in Art. 12, establishes that both sound and light are propagated in form of waves. Dr. Vincent obtained beautiful photograph of the interference phenomenon in 1898 formed by the 'ripples' on mercury surface produced by periodic disturbance of tuning-forks.

116. Experiments on Interference—(1) A direct way of demonstrating interference between two trains of sound waves is due to Quincke in 1886. A tube known as 'trombone' tube or Quincke's tube shown in fig. 50 consists of two arms T and T'. The length of the arm T' can be varied by pulling it out. The tube has two openings one of

* See Phil. Mag. 46, 1898.

which is connected to a small funnel S and the other is connected to the ear by a stethoscope. A source of sound emitting a pure tone of moderately high pitch, is held near the opening S. If the two arms are of the same length the sound waves from the source at S travel two paths STL and $ST'L$ of equal length and evidently reach the ear at

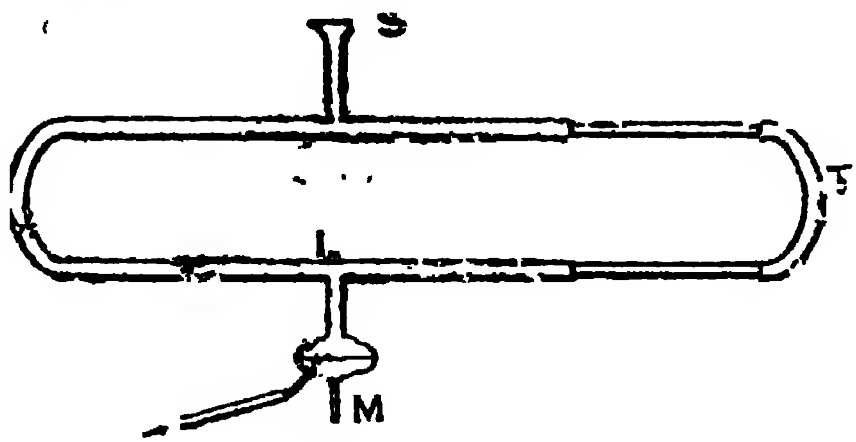

FIG. 50

the same phase. They, therefore, reinforce each other and a fairly loud sound is heard. If the tube T' is now gradually drawn out, the intensity of the sound reaching the ear will be reduced to a minimum when the difference in length of the two paths amounts to half the wave-length of the sound emitted by the source. The sound waves in this case reach the opening L at opposite phases and interfere with one another.

If the opening L is connected to a manometric flame as shown in the figure, the flame is disturbed to a maximum, as seen in a rotating mirror (Art. 114), when the two arms T and T' are equal in length. But the flame is not disturbed when the tube T' is drawn out such that the difference between the two paths T and T' amounts to half the wave-length of the sound emitted.

If the tube T' is further drawn out a point is reached

when the two paths differ by one wave-length. The wave trains from S reach the opening L at the same phase and the maximum disturbance of the flame results in. Thus a series of positions of T' will be obtained when the flame is most violently disturbed and for another series of positions of T' the flame rests. The first series corresponds to a path-difference zero or any multiple of the wave-length of the sound, between the two arms and the second , series corresponds to a path difference of any odd multiple of half wave-length of the sound between the two arms.

The apparatus can be applied to determine the velocity of sound in a tube, if frequency of the source is known and *vice versa.*

(2) *Interference from a Tuning-fork*—The tuning-fork provides the simplest illustration of the interference phenomena. If a small tuning-fork held by the stem is struck and whirled round near the ear, an intermittent sound will be heard : its sound being almost lost four times in each revolution.

When the prongs of the tuning-fork separate from each other, a wave of compression starts from the outer surface of each of the prongs A and B and proceeds in the direction of separation. At the same time a wave of rarefaction starts from the region C between the two prongs and proceeds in the direction perpendicular to their line of separation (fig. 51). Again when the prongs approach, a

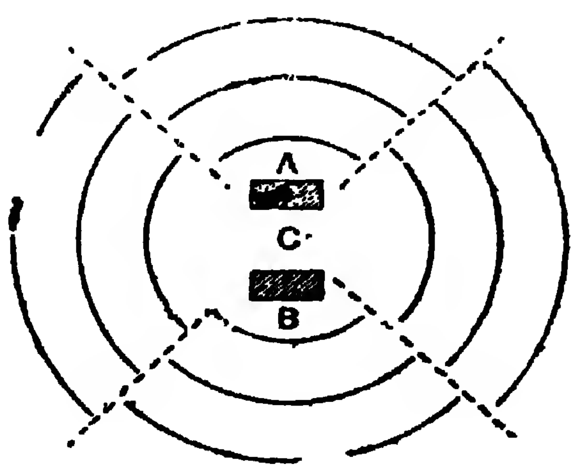

FIG. 51

wave of rarefaction proceeds in the direction of approach but a wave of compression proceeds in the direction perpendicular to their line of motion. The air particles in the intermediate directions (marked by the four dotted straight lines) are simultaneously acted upon by compressional and dilatational waves of equal amplitude from both sides, *i. e.* by two trains of waves of equal amplitude but of opposite phases. The waves thus interfere in these directions and produce silence.

One of the best examples of the interference of sound is the formation of stationary waves between the incident and reflected wave trains from a reflecting surface. The two wave trains are always at opposite phases at the node and give the node a permanent type. On the other hand, at the antinode they appear sometimes at the same phase and sometimes at opposite phases, at no other points they appear at the same phase ; so that the maximum displacements can take place at certain places only which are the seat of the antinodes. Interference is thus produced in the organ pipe, sonometer string, Chaldni's plate, etc.

117. **Beats**—When two sources of sound have exactly the *same* frequency, the waves from them reach somewhere in the space surrounding the two sources, at the same phase and somewhere at opposite phases. The intensity of sound at the former place is a maximum while that at the latter place is a minimum. At any other place the intensity has an intermediate value. With change of time there will be no change in the intensity at any particular place ; for, if at a particular place the compression due to one source superposes on the rarefaction due to the other, and produces silence, after half the periodic time of the either

source, the rarefaction due to the former will superpose on the compression due to the latter at the same place and produce silence. The waves from both the sources will thus arrive at opposite phases at this particular place at *all times* and produce silence. Thus when the two sources have precisely the same frequency, sounds of maximum or minimum intensity appear at *certain places throughout the time.*

If the two sources of sound have *slightly different* frequencies, a different state of affairs occurs. For, at any particular place the waves from the two sources will at one time be in the same phase when the two compressions or rarefactions superpose, but as time passes on, due to the unequal periods of the two sources, the quickly vibrating body gains half a vibration over the other so that the waves are at the opposite phases and they interfere with one another. Thus when the two sources of sound have slightly different frequencies, the intensity of the resulting sound at *any place* is maximum at *certain times* and minimum at *some other times.* This *periodic* variation in the *intensity* of the resulting sound constitutes the phenomenon of beats.

118. Frequency of Beats—When the two sources of sound having *nearly* the same frequencies beat with each other it can be shown both graphically and analytically that the number of beats produced per second is equal to the difference between the frequencies of the two sources.

(*a*) *Graphical Treatment*—In fig. 52, the displacement curves for two sound waves having equal amplitude but

frequencies 6 and 7 are drawn. Fig. 53 shows the resultant of the above two curves. It will be found that the amplitude of the resultant motion is not constant but varies periodically. This produces a periodic fluctuation in the intensity of the resulting sound. At A and C the two waves are at the same phase and assist each other while at B they are at opposite phases and cancel each other. The resulting curve within the length AC represents the

FIG. 52

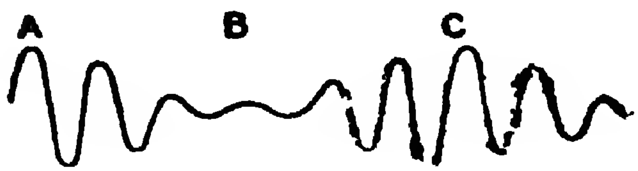

FIG. 53

combination of the two component wave trains in one second. The interval of time between two successive maximum amplitudes of the resulting sound is thus one second. The number of beats formed in one second is thus one. In general, the number of beats formed per second is equal to the difference between the frequencies of the tones producing them.

(*b*) *Analytical Treatment*—Let the displacement equations of the two waves be represented by

$$x_1 = a \cos mt \text{ and } x_2 = b \cos (m+n)t.$$

If n is supposed to be small compared to m, the frequencies of the two waves $\frac{m}{2\pi}$ and $\frac{m+n}{2\pi}$ are nearly equal.

Let the resultant displacement be given by

$$x = c \cos (mt + \varphi).$$

Since by the principle of superposition $x=x_1+x_2$, we have

$$c \cos (mt+\varphi)=a \cos mt+b \cos (m+n)t$$

or $c \cos mt \cos \varphi - c \sin mt \sin \varphi$

$$= a \cos mt + b \cos mt \cos nt - b \sin mt \sin nt.$$

As the above equality holds good at all times we can choose t such that $\cos mt$ vanishes. This evidently takes place when $t=\frac{\pi}{2m}$ so that both sides include terms containing $\sin mt$ only. Equating the coefficients of $\sin mt$

$$-c \sin \varphi = -b \sin nt \quad \ldots \quad \ldots \quad (1)$$

Similarly, choosing t such that $\sin mt$ vanishes when $t=\frac{\pi}{m}$, we have on both sides terms containing $\cos mt$ only and equating their coefficients

$$c \cos \varphi = a + b \cos nt \quad \ldots \quad \ldots \quad (2)$$

Squaring and adding (1) and (2)

$$c^2 = a^2 + b^2 + 2ab \cos nt$$

$$\text{whence } c = \sqrt{a^2+b^2+2ab \cos nt} \quad \ldots \quad (3)$$

Again from (1) and (2), $\tan \varphi = \dfrac{b \sin nt}{a+b \cos nt} \quad \ldots \quad (4)$

Equation (3) shows that c, the amplitude of the resultant motion is not constant but fluctuates periodically with time. This fluctuation in amplitude produces a periodic variation in the intensity of the resulting sound which is known as the phenomenon of beats.

We find from relation (3) that c is a maximum if $\cos nt=1$ or $nt=0, 2\pi, 4\pi, 6\pi$, etc.

$$\text{or } t=0, \frac{2\pi}{n}, \frac{4\pi}{n}, \frac{6\pi}{n}, \text{ etc.}$$

The interval between any two consecutive maximum values of c is $\frac{2\pi}{n}$. The maximum value of $c=a+b$.

Similarly, c is a minimum if $\cos nt = -1$ or $nt=\pi, 3\pi, 5\pi$, etc.

$$\text{or } t=\frac{\pi}{n}, \frac{3\pi}{n}, \frac{5\pi}{n}, \text{ etc.}$$

The interval between any two consecutive minimum values of c is $\frac{2\pi}{n}$. The minimum value of $c=a\sim b$ (the difference between a and b).

Thus the period of fluctuation in amplitude (or intensity) of the resulting tone which is the time between two consecutive maximum or minimum values of it $=\frac{2\pi}{n}$.

Hence the frequency of fluctuation in *intensity* of the resulting tone which is the frequency of beats $=\frac{n}{2\pi}$. Again, the difference between the frequencies of the component tones $=\frac{m+n}{2\pi}-\frac{m}{2\pi}=\frac{n}{2\pi}$.

Thus the frequency of beats is equal to the difference between the frequencies of the tones producing them.

If the amplitudes of the component tones are equal $b=a$, that of the resulting tone varies between 0 and $2a$.

It will be found that the *frequency* of the resultant vibration is $\frac{m}{2\pi}$. * Equation (4) shows that the phase angle

* This is true if n is very small compared to m. If n is comparatively large, but still lies within the range of formation of beats, the resulting displacement is represented by

$$z=c\cos\left\{\left(m+\frac{n}{2}\right)t+\phi\right\}$$

φ of the resultant motion is not a constant but varies with time being dependent on t.

The fact that the frequency of beats is equal to the difference between the frequencies of the component vibrations, forms a ready method for measuring small differences of frequencies or the frequency of one vibrating source when that of the other is known. For example, two tuning-forks of nearly equal frequencies, one of which is of known frequency, are sounded together and the frequency of beats they produce is determined by a stopwatch by noting the time taken to produce a certain number of beats. The prong of one of them is then loaded by a *little* wax ; this decreases its frequency of vibration. The two forks are then sounded together and the frequency of beats again counted in the above way. If the frequency of beats be

so that the frequency of the resulting vibration is the arithmetic mean between those of the component vibrations. Proceeding in the same way it will be found that

$$c^2 = a^2 + b^2 + 2ab \cos nt$$

which shows that the frequency of beats is equal to the difference between the frequencies of the component tones.

It will be found that the above deduction will hold good if the difference in frequencies of the component tones were not small, but the assumption of the resultant vibration in the form assumed is not strictly true, when n is comparable to m. Lord Rayleigh observes,—

"As the interval (ratio of the frequencies) between the component tones increases, the component tones become more and more prominent, and the beats diminish in loudness and distinctness till by the time a certain interval is reached, which is about a minor third (Art. 180) in the middle of the scale, the beats practically disappear and the two tones alone survive.", *Vide* 'Theory of Sound', Lord Rayleigh, Vol. II p. 444.

less than the previous value, the fork, which is loaded, has the larger frequency of the two. If, on the other hand, the frequency of beats be greater than what it was originally, the loaded fork has the smaller frequency of the two.

Since the 'beating' effect is due to the fact that at certain equal *time* intervals the wave trains agree in phase and reinforce each other whilst at certain intermediate *times* they are opposite in phase and cancel each other, the phenomenon of beats may be regarded as an example of interference.

119. Combination Tones—It was discovered independently by Sorge, a German organist, in 1745 and more precisely by Tartini, an Italian violinist, in 1754 that when two tones of frequencies n_1 and n_2 superpose, a third low tone of frequency $n_1 - n_2$ was produced. This new tone, called Tartini's tone after the name of its discoverer, was subsequently produced by two forks of moderately high frequency having a difference of frequencies near about fifty. König, in 1899, produced them from the longitudinal vibrations of two glass rods.* Helmholtz further examined the matter fully and gave the name *difference tone* to them from the fact that the frequency of the tone was equal to the difference between the frequencies of the two tones producing them.

120. Beat Tone Theory—Soon after Tartini's discovery it was suggested by Lagrange in 1759 and later by Young in 1800 that the *difference tones* were the same as beats. When the beats occurred with sufficient rapidity,

* Professor Barton gives details of some methods of producing these tones. See 'A Text-Book of Sound,' Barton, Arts. 298-301.

the ear recognised them as a new tone. This is known as the *beat tone theory* or König's theory according to which the beats and the difference tones were physically identical. It is the ear which distinguishes between them recognising more than 16 beats per second as a new tone.

Helmholtz later on carried out an extensive work on this and discovered that along with the difference tone another tone having a frequency $n_1 + n_2$ or the sum of the frequencies of the component tones was produced. This he called the *summation tone*. He subsequently discovered tones having frequencies $2n_1$ and $2n_2$ which were the double of those of the primaries. These he called the *self-combination tones* and all the new tones produced were generally called *combination tones*. It was further observed by him that the combination tones were produced only when the primary tones were intense and that feeble primaries failed to produce them. This led to the discovery of the Intensity Theory of combination tones by Helmholtz.

121. Intensity Theory—Helmholtz assumes that the displacement x of a system capable of vibration under the action of a force f is of the form

$$f = ax + bx^2 + cx^3 + \dots\dots\dots$$

For small displacements, x^2, x^3, etc., are small compared to x and the variation is of the linear form

$$f = ax$$

If a system is separately subjected to two forces, f_1 and f_2, the forces being sufficiently intense to produce large displacements x_1 and x_2 respectively, the displacement equations can be put down as

$$f_1 = ax_1 + bx_1^2 + cx_1^3 + \dots \qquad \dots \qquad (1)$$

and $$f_2 = ax_2 + bx_2^2 + cx_2^3 + \dots \qquad \dots \qquad (2)$$

If the two forces act simultaneously on the system producing a displacement X,

$$f_1+f_2=aX+bX^2+cX^3+\ldots \qquad \ldots \qquad (3)$$

From (1) and (2),

$$f_1+f_2=a(x_1+x_2)+b(x_1{}^2+x_2{}^2)+c(x_1{}^3+x_2{}^3)+\ldots(4)$$

A comparison of equations (3) and (4) shows that X is not equal to the sum of the component displacements x_1 and x_2.

If, on the other hand, the component displacements x_1 and x_2 are small, the terms containing $x_1{}^2$, $x_1{}^3$, $x_2{}^2$, $x_2{}^3$, etc., are small compared to the linear terms, and the equations (1) and (2) reduce to

$$f_1=ax_1 \text{ and } f_2=ax_2$$
$$\text{so that } f_1+f_2=aX=a(x_1+x_2)$$
$$\text{and} \quad X=x_1+x_2$$

In other words, if a system is subjected to two *intense* forces producing large displacements, the resultant displacement is not equal to the simple sum of the component displacements, *i. e.* the principle of superposition does not hold good in the system (Art. 27); such a system is called an *asymmetric system.* Helmholtz worked out the effect of subjecting an asymmetric system to two forces and successfully explained the formation of the difference, summation and self-combination tones. * The mathematics of Helmholtz showed that the intensity of the difference tone was greater than that of any other combination tone and because of this the difference tone was observed first.

See 'Dynamical Theory of Sound', Lamb, Art, 95.

122. Objective and Subjective Combination Tones— The beat-tone theory of combination tones suggests that the formation of combination tones was more a physiological effect than a physical phenomenon. It is the ear that recognises the quick succession of beats as a distinct tone which was formed within the ear and thus only had a subjective existence. It was supposed that they were not formed in the external medium and thus had no objective reality. Helmholtz ultimately assured himself of the objective existence of combination tones by the sympathetic resonance of membranes and air resonators tuned in unison with his double siren producing the combination tones. His experiments with harmonium also revealed that they had a subjective existence. The harmonium was provided with foot bellows as well as reserve bellows which could be pumped full and connected to or disconnected from the harmonium by a stop. The combination tones were first produced by working both the bellows, but when the reserve bellows was cut off he could still recognise the combination tones although they were weaker than before. This is due to the fact that when the reserve bellows was stopped, the objective portion of the combination tone could not be formed as there were no longer two forces to produce them. It was only the subjective portion of the combination tones perceived.

The experiment of Professor Rücker and Edser with the Michelson interferometer in 1895 conclusively proved the objective existence of the combination tones. One of the reflecting mirrors in the interferometer was mounted on one prong of a tuning-fork of frequency 64, which acted as a resonator to one of the combination tones. The generating

tones were produced by a Helmholtz double siren. It is relatively difficult to excite a tuning-fork by resonance, but the arrangement was so delicate that a movement of 1/80,000 of an inch would alter the length of the path of one of the interfering rays by one wave-length. Taking adequate precautions to eliminate external disturbances, when two tones having a difference of frequency 64 were produced by the double siren, the bands shifted in the field of view which shows the formation of a new tone of frequency 64 to which the tuning-fork responded. This conclusively proves the objective existence of combination tones.

123. Waetzmann's General Asymmetry Theory— It will be observed that the beat tone theory suggests the subjective formation and the intensity theory explains the objective formation of combination tones. Waetzmann's general asymmetry theory propounded in 1920 of the combination tones reconciles the beat tone theory and the intensity theory and explains the subjectivity of the combination tones. He took a membrane and loaded one side of it with a central weight. The membrane was subjected to two periodic disturbances from two tuning-forks and the displacement curve of the membrane was recorded by reflecting a narrow beam of light from it. Performing the Fourier analysis of the curve, Waetzmann finds the original tones, a difference tone and occasionally a summation tone. The drum skin of the ear being centrally loaded on one side by bones represents an asymmetric system capable of producing the combination tones.

Examples

1. Explain by drawing suitable graphs, the formation of beats when two notes of nearly the same pitch are produced. Prove that the

frequency of the beats is equal to the difference of the frequencies of the given notes. What is the effect of the beats on the concordance of musical notes? (*C. U. 1919*)*

2. Define pitch and quality of a musical note. Explain how they are represented in the wave curve. Explain graphically or mathematically the formation of beats when two musical notes of nearly equal pitch are sounded together. (*C. U. 1922*)

3. Explain and illustrate by diagram the occurrence of beats when two forks of nearly the same pitch are sounded together. How would you determine which fork is vibrating faster? (*C. U. 1929*)

4. Write short notes on the following :—(*a*) Interference of sound waves, (*b*) beats and (*c*) combination tones. How can the existence of these phenomena be shown? (*C. U. 1930*)

5. Explin by means of a diagram, the production of beats. (*C. U. 1934*)

6. Describe an experiment to illustrate the interference of sound waves.

An open organ pipe produces 8 beats per second when sounded with a tuning fork of 256 vibrations per second, the fork giving the lower tone. How much must the length of the pipe be altered to bring it into accord with the fork? (Velocity of sound in dry air = 286 metres per second.) (*C. U. 1939*)

7. Explain the formation of beats when two notes are sounded together. How do you account for consonance and dissonance in combinational tones?*

Sixty-four tuning forks are arranged in order of increasing frequency and any two successive forks give four beats per second when sounded together. If the last fork gives the octave of the first, calculate the frequency of the latter.

8. Discuss the phenomenon of interference in case of sound and explain the formation of beats. Two tuning-forks *A* and *B* are sounded together and 8 beats per second are heard. *A* is in resonance with a column of air, 32 cm, long in a pipe closed at one end and *B* is similarly in resonance when the length of the column is increased by one centimetre. Calculate the frequency of the forks. (*C. U. 1950*)

9. Explain the formation of beats when two notes are sounded together. An organ pipe produces 8 beats per sec. when sounded with a fork of 256 vibrations per sec., the fork giving the lower note. How much must the length of the pipe be altered to bring it in unision with the fork? (*C. U. 1953*)

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* See Chapter XIV.

CHAPTER XIV

CONSONANCE, DISSONANCE AND MUSICAL SCALE

124. Consonance and Dissonance—It is a matter of common experience that the simultaneous sounding of two or more notes from a musical instrument often produces a harmoniousness or pleasant sensation to the ear. The notes are said to produce consonance or concord. On the other hand, when two or more notes sounded together produce a disagreeable sensation to the ear, they are said to produce dissonance or discord.

Before the Christian era the Greeks had studied the musical consonance and dissonance. They found that the different lengths of the string of a stringed musical instrument which produced consonance bore a simple relation among them, such as 1 : 2, 2 : 3, and so forth. On the other hand, lengths in the ratio 64 : 81 among which there was no simple relation produced dissonance. Pythagoras made the noteworthy observation that the sounds produced by the two segments of a string into which it was divided were more and more consonant the simpler the ratio of the lengths of the two segments.

125. In 1700 Sauveur recognised that octaves and other sounds having simple ratio among their frequencies were concordant because the beats formed by them were too quick to be distinguished. In 1862 Helmholtz published his eight years' work on acoustics and gave a masterly insight into the subject. Helmholtz's theory in its bare outline was that all dissonances or discords were due to

unpleasant beats formed by the component notes. Consonances or concords are formed by two notes which fail to produce the unpleasant beats.

126. Helmholtz's view as to why beats should be unpleasant at all is that the effect of beats to the ear is analogous to that of flickering light to the eye. Beats constitute a periodic variation in intensity of the resultant of two tones. During the loudest phase of the beats the ear gets somewhat fatigued and during the feeblest phase the ear is rested and its sensibility restored ; the recurrence of the loudest phase in this specially sensitive state of the ear is distressing which produces the disagreeable sensation or dissonance.

127. In Helmholtz's view the discord due to a certain number of beats is not the same at all parts of the musical scale. Thus 33 beats per second produces the harshest dissonance in the neighbourhood of *C* of the musical scale. The dissonance arising out of the sounding of two notes of a musical scale depends in a compound manner on the magnitude of the interval between the two notes (ratio between their frequencies) and on the frequency of beats formed by them. The beats to produce the dissonance may be produced by the two component notes or by the overtones with which the primary tones produced by a musical instrument are associated. Thus the dissonance produced by two notes depends to a large extent upon the quality of the notes. Besides this, if the primary tones be forcible enough to produce combination tones, the dissonance may be produced by the beats formed between any of the combination tones and one of the primaries, or between two combination tones formed.

128. The adjoining curve (fig. 54) is a reproduction of Helmholtz's estimation of the degree of discord plotted as ordinates against the intervals within an octave plotted as abscissæ. If two notes are originally tuned to unison (C') and then while one is kept at that pitch and the other gradually raised to one octave higher, the conditions will be as represented in the diagram. It will be found that all the well-known intervals (Art. 130) are represented by dips of the curve and are more or less closely bounded by strong dissonance.

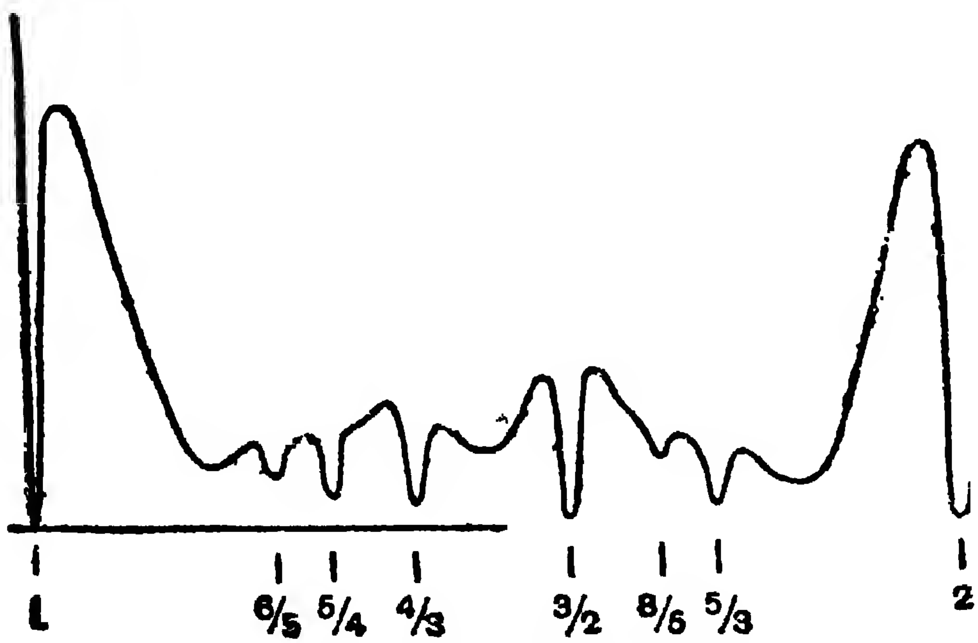

FIG. 54

129. Musical Scale—In a musical instrument such as piano, harmonium, orchestral wind instrument, etc., a fixed number of notes are produced by the keys the instrument is provided with. In an instrument like a violin any number of notes can, however, be produced by the skill of the operator to suit a vocal music. The object of a musical scale is to determine the particular notes to be associated with the keys in the former class of instruments within an octave to be most suitable to the vocal music.

130. Musical Interval—The absolute frequencies of the notes in a musical scale are immaterial. These vary in different countries and have changed considerably in the course of time. In passing from one musical note to another, the ear recognises the *ratio* in which their frequencies altar. The ratio between the frequencies of two notes is called the *interval* between the two. If two notes having frequencies m and n change in frequencies but retain the same ratio or interval m/n between them, the ear hardly recognises any distinction in passing from one note to the other.

It has already been said that if the ratio between the frequencies of two notes can be expressed in the ratio of small whole numbers, the two notes form consonance or harmonious effect. The interval between the two notes is called *consonant interval.* If, on the other hand, the interval between two notes cannot be given in the ratio of small whole numbers, the notes produce dissonance and the interval between them is called a *dissonant interval.*

It is needless to say that the unison (1 : 1) is the most consonant interval, next comes the octave (2 : 1). The following list shows the names and the intervals (*consonant intervals*) arranged in order of their consonant effect to the ear.

Unison	...	1 : 1	Major Third	...	5 : 4
Octave	...	2 : 1	Minor Third	...	6 : 5
Fifth	...	3 : 2	Major Sixth	...	5 : 3
Fourth	...	4 : 3	Minor Sixth	...	8 : 5

131. Diatonic Scale—The musical scale which survived for a considerable time from time unknown is the *diatonic*

musical scale of diatonic scale. The scale comprises eight notes between the lowest of the series called the *tonic* or *keynote* and its *octave.* The names of the notes commencing from the tonic are *Do, Re, Mi, Fa, Sol, La, Ti, Do.* According to the notation introduced by Helmholtz, the notes are respectively denoted by the letters C, D, E, F, G, A, B, c. A musical instrument cannot be complete with these few notes only and the notes which are respectively one octave higher than the above are denoted with small letters as, c, d, e, f, g, a, b, c^1. The next higher scale is denoted with one dash above the small letters, the next higher one with two dashes and so on. The scale lower than C—c is denoted with suffix 1 below the capital letters as, C_1 D_1 E_1, F_1, G_1, A_1, B_1, C. The next lower one with the suffix 2 and so on.

The following table shows the notes on a diatonic scale in the first line, with Helmholtz's notation in the second line, and the relative frequencies among them in the third line taking the frequency of the tonic as unity. The fourth line shows the interval between any two consecutive pair and the last line gives the smallest integers proportional to the frequencies.

Do,	*Re,*	*Mi,*	*Fa,*	*Sol,*	*La,*	*Ti,*	*Do*
C	D	E	F	G	A	B	c
1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	2
	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$
24	27	30	32	36	40	45	48

Commencing from C, the tonic, the notes in the scale are usually referred to as the first, second, third, fourth, fifth, sixth, seventh and octave respectively. It will be noticed that the interval between consecutive pairs is not he same throughout the scale but there are three distinct intervals $\frac{9}{8}$, $\frac{10}{9}$ and $\frac{16}{15}$. The intervals $\frac{9}{8}$ and $\frac{10}{9}$ are called *tones* of which the former is called a *major tone* and the latter a *minor tone*. The interval $\frac{16}{15}$ is called a *semitone* or *limma*. The interval between a major tone and a minor tone $=\frac{9/8}{10/9}=\frac{81}{80}$, called a *comma*.

132. Advantages and Disadvantages of the Diatonic Scale—The origin of the diatonic scale which has been in use amongst European nations for many centuries is almost unknown. It seems probable that the scale got its development from a study of the consonance and dissonance produced by the sounding of notes in vibrating string with which the ancients were acquainted, as there is no doubt that the reason for its long survival is that no other scale designed up till now is so well fitted to provide harmonious combinations. This is evident from the fact that the various intervals are expressed in terms of small whole numbers (Art. 130) save and except the two intervals between D and C and between B and C which are more or less dissonant.

An outstanding defect of the diatonic scale lies in its 'power of modulation.' A characteristic of modern music is that it frequently modulates or changes its tonic. If a music is composed on the diatonic scale with C as tonic and

a frequency 256 is assigned to it, the frequencies of the other notes in the scale will be as follows :

C	*D*	*E*	*F*	*G*	*A*	*B*	*c*	*d*	*e*
256	288	320	341·3	384	426·7	480	512	576	640

Let us see if the same music can be composed with *E* as tonic on the above scale. If composed with *E* as tonic the music requires the following frequencies :

E	*F*	*G*	*A*	*B*	*c*	*d*	*e*
320	360	400	426·7	480	533·3	600	640

It will be found from a study of the above that the scale used for composing the music with *C* as tonic does not contain all the notes needed for composing the music with *E* as tonic, but requires a large number of new notes to be added to it. Similarly to compose the music with another note as tonic a number of other new notes will be necessary. Thus a diatonic scale with one note as a tonic cannot serve with a different note as the tonic. It is obviously because of the fact that the interval is continually changing which is expressed by stating that the modulation of the diatonic scale is limited. This gave rise to other scales known as 'tempered scales.'

133. **Temperament**—The fixing of a suitable number of notes between tonic and its octave is a compromise. For, in designing the scale we shall have to look to (1) the power of modulation of the scale which necessitates a large number of notes within the octave rising in pitch very gradually, (2) practical convenience,—according to which a large number of notes within the octave interferes with the simplicity of construction of the musical instrument and brings in difficulty in playing upon it. (3) Further, to

satisfy the cultivated ear the simultaneous sounding of two or more notes should as far as possible be free from dissonance.

A compromise of these claims, some of which are conflicting, is called *temperament.* It fixes the number of notes between a tonic and its octave to twelve. The scale constructed as a result of the compromise is called a *tempered* scale.

134. Equal Temperament— Various systems of temperament of historical importance have been used, the simplest one which is now most commonly used or at least aimed at is the *equal temperament* ; the scale with this temperament is called the *equi-tempered* or *chromatic scale.* The temperament divides the interval between the tonic and its octave into twelve equal intervals. Hence, if x be the interval between two consecutive notes, we have

$$x.x.x....(12 \text{ factors})=2$$
$$\text{or } x^{12}=2$$
$$\therefore \quad x=2^{\frac{1}{12}}=1\text{·}05946.$$

One additional black key is placed in piano or harmonium dividing each of the five *tones* (the intervals $\frac{9}{8}$ and $\frac{10}{9}$ are called *tones,* see art, 131) in the diatonic scale given by white keys. The notes with their relative frequencies are given in the following table.

C	$C\sharp$	D	$D\sharp$	E	F	$F\sharp$	G	$G\sharp$	A	$A\sharp$	B	C
1	$2^{\frac{1}{12}}$	$2^{\frac{1}{6}}$	$2^{\frac{1}{4}}$	$2^{\frac{1}{3}}$	$2^{\frac{5}{12}}$	$2^{\frac{1}{2}}$	$2^{\frac{7}{12}}$	$2^{\frac{2}{3}}$	$2^{\frac{3}{4}}$	$2^{\frac{5}{6}}$	$2^{\frac{11}{12}}$	2

The white keys in the piano or harmonium correspond to original tones C, D, E, F, G, A, B, c in the diatonic scale.

The note $C\sharp$ is called C sharp or C sharpened. It is also written as $D\flat$ or D flat, *i.e.* D flattened. Similar is the notation $D\sharp$. $F\sharp$. $G\sharp$. and $A\sharp$. The additional tones are given by black keys.

Examples

1. Explain what you understand by the musical interval between two notes. What intervals are used in the diatonic major scale? What is temperament, and why is the tempered scale used in the keyed instruments? (*C. U. 1932.*)

2. Explain fully what you mean by a musical scale and the interval between the musical notes. State briefly how the intervals in a diatonic scale are classified. (*C. U. 1935*)

3. Explain the formation of beats when two notes are sounded together. How do you account for consonance and dissonance in combinational tones?

What is the velocity of sound in a gas in which two waves of length 1 and 1·01 metres respectively produce 10 beats in 3 seconds?

(*C. U. 1936*)

4. What is concord and discord? Discuss the relationship between the frequencies of two notes which cause them to be discordant when sounded simultaneously. (*C. U. 1940*).

5. What is concord and discord? Discuss the relationship between the frequencies of two notes which cause them to be discordant when sounded simulataneously. (*C. U. 1951*)

6. Explain what you understand by the musical interval between two notes. What intervals are used in the diatonic major scale?

(*C. U. 1952*)

7. How are musical intervals measured? What is the physical cause of consonance and dissonance? (*C. U. 1953*)

CHAPTER XV

MUSICAL INSTRUMENTS

135. In this chapter a broad outline of some of the commonly used musical instruments is given from acoustical standpoint. For a fuller treatment some higher text-book on the subject such as Barton's Text-Book of Sound, should be consulted. The great variety of musical instruments can be broadly divided into three classes, according to the mode in which the musical note is produced by them. These are (1) *wind instruments* such as organ pipe, harmonium, etc, in which the note is produced by the vibration of air enclosed within the instruments, (2) *stringed instruments* such as guitar, pianoforte, violin, etc, in which the note is produced by the vibration of strings kept under tension and (3) *instruments of fixed pitch* such as kettle-drum which is used as a rhythm rather than a musical instrument in the proper sense.*

136. **Wind instruments, Organ Pipes**—There are again a large number of sub-classes of the wind instruments according to the nature of exciting a musical note in them. The two broad subdivisions of the wind instrument are,—(1) instruments without reeds, such as flute, piccolo, etc and (2) instruments with reeds, such as harmonium, clarionet, etc. The organ pipe which forms a familiar example of the wind instrument may be of the above two types,—(1) the one, in which there is no reed and is known as the **flue pipe** and (2) the other, in which the note is generated by the vibration of reeds and is called the **reed pipe.** The modes of production of the note in a flue pipe and in a reed pipe are as follows.

*See Art. 80.

137. The Flue Pipe—Fig. 55 shows a sectional diagram of a flue pipe. To maintain the vibrations of a cloumn of air in a tube, a blast of air from a jet O is directed to a sharp edge E at one end of the tube. A tone arises when the edge is at certain minimum distance from the jet. This is known as the 'edge tone'. Schmidtake in 1919 has shown that this critical minimum distance is dependent on the velocity of the gas issuing from the jet. The frequency of the tone falls off inversely as the distance of the edge from the jet. At a certain value of the distance, however, the frequency jumps to the octave higher and then falls again with increase of distance.

FIG. 55

The early view of the note emitted by the organ pipe was that the issuing laminæ of air vibrated transversely like a reed at a frequency governed entirely by the column of air. This is rejected at present. The modern view, confirmed by the experiments of Strouhal (1878), Kohlrausch (1881) and Wachsmuth (1904) is that the air emerging from the jet forms alternately spaced vortices on each side of the air stream. At a certain minimum blowing pressure impulses due to these vortices constituting the edge tone are of such a frequency that resonance is set up in the air column and the pipe 'speaks' with its fundamental tone. The higher harmonics of the pipe are excited by increasing the blowing pressure.

138. The Reed Pipe—In the other form of the organ pipe, known as the reed pipe, the air blast impinges on a flexible strip of metal called reed which controls the amount

of air entering the pipe by wholly or nearly covering an aperture through which the air passes. If the reed completely covers the aperture it is called a *beating* or *striking* reed (Fig. 56) but if it oscillates, nearly but not fully closing it, it is called a *free* reed (fig. 57). A beating reed is aways curved outwards so that when the air pressure closes it, the closing is done gradually from the fixed to the free end and not suddenly. This is done to avoid the harshness of the sound which occurs if the reed closes the aperture throughout its length at a time.

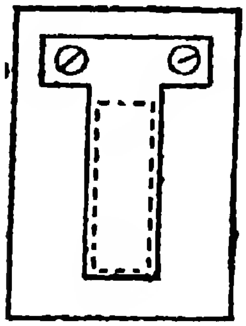

FIG. 56.

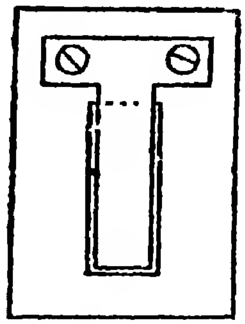

FIG. 57

Free reeds are now no longer used in organ pipes. They are used in harmonium, American organs, etc. The reed in an organ pipe and the pipe are regarded as a coupled system tuned to the same frequency of which the reed is in transverse vibration and the air column in longitudinal vibration. The air blast sets the reed in vibration and puffs of air are admitted to the pipe which is thereby set into resonant vibration.

FIG. 58

In the reed pipe. (fig. 58) the reed is tuned by means of a wire *W* which can be pushed towards the fixed or free end of the reed. If pushed towards the free end, the motion is restricted so that the effective length of the reed decreases and the frequency rises.

Similarly, if pushed towards the fixed end, the effective length increases and the frequency falls.

In some pipes such as cornet and horn, the lips of the operator serve the purpose of the reed and maintain the vibrations of the air column.

139. **Closed and Open Organ Pipes**—An organ pipe may be closed at one end and open at the other when it is called a *closed pipe.* It may also have both ends open when it is called an *open pipe.*

The behaviour of a closed end is that it is the seat of a node of the stationary waves set up in the organ pipe as the air particles in contact with this end can hardly be displaced from their normal positions. They are simply subjected to variations of pressures (Art. 88a). On the other hand, an open end is the seat of an antinode as the air particles here can have their free displacements both ways when subjected to alterations of pressure. (Art. 88b).

In a flue pipe *the end at which the jet is applied behaves as an open end* of the pipe as the air particles here will have free displacements due to pressure variation. In a reed pipe a *beating reed** *behaves as a closed end.* For when a pulse of rarefaction meets it, it closes the end and behaves like a fixed wall (see Art. 88*a*) and the pulse is reflected without change of type. Similarly, when it is the centre of compression it opens for a moment and admits a stream of air at high pressure which increases the condensation already existing. Thus a pulse of compression is reflected without any change of type which gives it properties of a rigid wall (Art. 88*a*).

* The behaviour of a free reed, which is no longer used in organ pipes, is not fully understood.

In 1847 Wertheim determined the velocity of sound propagation in liquids by organ pipes. The pipes were provided with suitable mouth-pieces and were immersed in the liquid. To make the tubes 'speak,' wherein lay the chief difficulty of the experiment, the tubes were injected with a stream of the liquid. The frequency of the note emitted was determined by any of the methods of finding the frequency suitable for the purpose. The wave-length was calculated from the length of the tube and the velocity found out. Wertheim found that the velocity of sound in pipes when filled with water was 1173 metres per second. This value which was much below that found in water when in large bulk (Art. 62) was interpreted by Wertheim by supposing that such an isolated cylinder of liquid increased in cross-section on the passage of condensation and contracted during the passage of a rarefaction. Wertheim introduced a correction for this and the corrected value agrees fairly with the experimental result of Colladon and Strum.

140. Fundamental and Overtones of Closed Pipe.—When the air column in a closed pipe vibrates so as to emit its fundamental tone, there is one node at the closed end and one antinode at the open end; there being no other node or antinode within the pipe as shown in fig. 59(*a*.) Neglecting the correction amounting to ·6r at the open end (see Art. 96) the

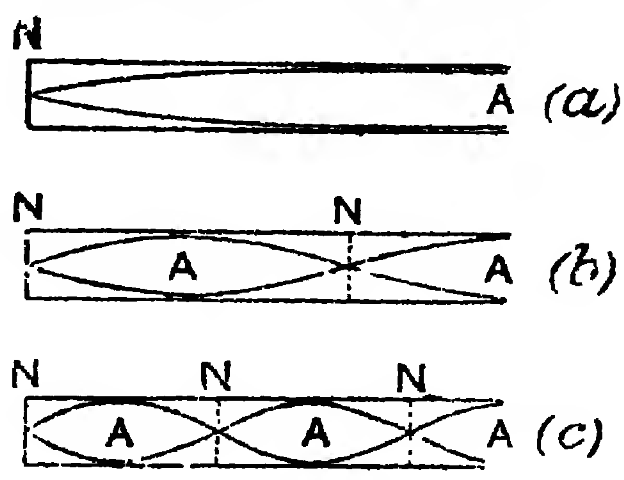

FIG. 59

length of the pipe amounts to that of one quarter wave, *i. e.* $\frac{\lambda}{4}$, where λ is the wave-length of the stationary waves within the pipe. Hence $\lambda=4l$, where l=the length of the pipe. The frequency of the fundamental tone is thus given by

$$n=\frac{v}{\lambda}=\frac{v}{4l}\ldots \quad \ldots \quad (1)$$

(where v = the velocity of sound waves in air).

In the next higher tone possible, the mode of vibration of the pipe is as shown in fig. (*b*). The open and closed ends being the seats of an antinode and a node respectively, there will be one additional node and one additional antinode within the pipe. The pipe evidently contains three quarter-waves. The length l of the pipe is given by

$$l=\frac{3}{4}\lambda \quad \text{or} \quad \lambda=\frac{4}{3}l$$

If the frequency of vibration of the pipe is n_1

$$n_1=\frac{v}{\lambda}=\frac{v}{\frac{4l}{3}}=\frac{3v}{4l}=3n\ldots\text{(from 1)}$$

In the next higher tone which the pipe can emit, it will have two intermediate nodes and two intermediate antinodes besides the node at the closed end and antinode at the open end as shown in fig. (*c*). The pipe then contains five quarter-waves. Hence the length l of the pipe is given by

$$l=\frac{5}{4}\lambda \quad \text{or} \quad \lambda=\frac{4}{5}l$$

and the frequency of vibration n_2 is given by

$$n_2=\frac{v}{\lambda}=\frac{v}{\frac{4l}{5}}=\frac{5v}{4l}=5n\ldots \text{ (from 1)}$$

It can similarly be shown that the other higher modes of vibration will yield tones of frequencies $7n$, $9n$, etc.

Thus the possible tones of a closed pipe have frequencies proportional to *odd natural numbers*.

141. Fundamental and Overtones of an Open Pipe—When the air within an open pipe vibrates so as to emit its fundamental tone, each open end is the seat of an antinode and there is one node in the intermediate position as shown in fig. 60(*a*). Neglecting the corrections at the open ends (Art. 96) the tube includes one half-wave.

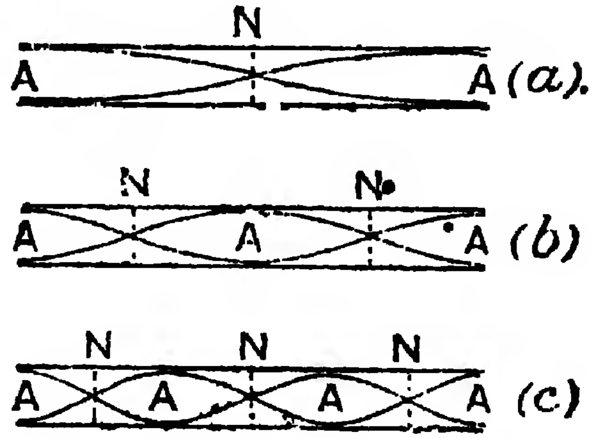

FIG. 60

The length l of the tube is thus given by

$$l = \frac{\lambda}{2} \text{ or } \lambda = 2l, \text{ where } \lambda \text{ is the wave-length}$$

of the stationary vibration within the pipe.

The frequency of the fundamental tone is thus given by

$$n = \frac{v}{2l} \dots \quad (1)$$

where v = the velocity of sound waves in air.

In the next higher tone possible, the mode of vibration of the pipe will consist of two nodes and one antinode within the pipe besides the two antinodes at the open ends, as shown in fig. (*b*). The pipe then encloses four quarter-waves, and its length l amounts to λ.

The frequency of the tone emitted is given by

$$n_1 = \frac{v}{\lambda} = \frac{v}{l} = 2n \dots \text{ (from 1)}$$

In the next higher tone which the pipe can emit, it will include three intermediate nodes and two intermediate

antinodes besides the two antinodes at the open ends. The pipe then includes six quarter-waves and its length l is given by

$$l = \frac{6\lambda}{4} = \frac{3}{2}\lambda \quad \text{or} \quad \lambda = \frac{2}{3}l$$

The frequency of the tone n_2 is given by

$$n_2 = \frac{v}{\lambda} = \frac{v}{\frac{2l}{3}} = \frac{3v}{2l} = 3n \ \dots \ \text{(from 1)}$$

It can similarly be shown that the other higher modes of vibration will yield tones of frequencies $4n$, $5n$, etc.

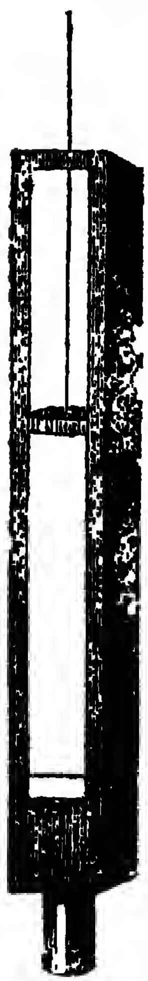

FIG. 61

Thus the possible tones of on open pipe have frequencies *proportional to the natural numbers.*

The presence of all the harmonics in the open pipe makes the note emitted by it much richer and fuller than that emitted by a closed pipe.

A study of the frequencies of the fundamental tones emitted by a closed and an open organ pipe shows that if the pipes are *identical*, the frequency of the fundamental tone of the open pipe is *one octave higher* than that of the fundamental tone emitted by a closed pipe.

142. Experiments on the Phenomena within an Organ Pipe—One of the earliest experiments of observing the relative displacements in different parts of an open pipe is due to Savart. One side of the organ pipe has a glass wall. A light ring (fig. 61) is covered on one side with a tightly stretched paper or thin membrane on which a little fine dry sand is sprinkled. The ring is suspended by light cords and lowered into a pipe in

vibration. At an antinode where there is a maximum motion and displacement of air particles, the membrane with the sand on it is violently agitated. The motion of the sand can be observed through the glass wall and the rattling sound produced by the striking of the sand on the membrane distinctly heard. At the node where there is no motion or displacement of the air, the sand particles are at rest and no rattling sound is heard. At any intermediate place the motion of the sand or the rattling sound is intermediate. The method is used in locating the positions of intermediate nodes and antinodes within the tube and in finding the overtones produced by it.

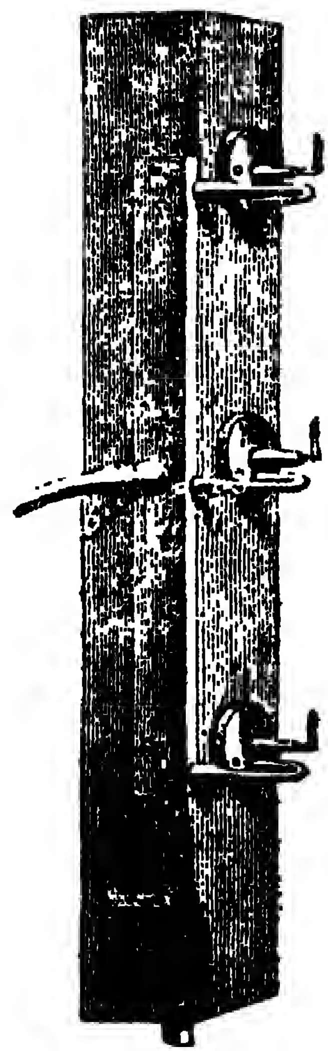

FIG. 62

The positions of the intermediate nodes and antinodes within a closed or an open organ pipe are studied by the variations of pressure at these points with the help of König's manometric capsule described in Art. 114. The organ pipe is fitted with manometric capsules fed by one supply tube (fig. 62). The behaviour of the burners can be observed by a rotating mirror.

When a manometric capsule is fixed at an antinode, where there is hardly any pressure variation, the flame is steady as seen in the mirror. A manometric capsule fixed at a node is subjected to a maximum pressure variation and the flame jumps up and down with the maximum amplitude and with a frequency equal to that of the air column. At any intermediate point the pressure

variation is intermediate ; there is also periodic disturbance of the flame but with less amplitude than at a node.

Richardson's manometric membrane described in Art. 114 is a more modern arrangement for locating the nodes and antinodes.

Töpler and Boltzmann in 1870 devised an optical method* based on Jamin's optical interferometer to examine the change of density of the air within an organ pipe and to locate the positions of nodes and antinodes thereby.

143. Effect of Temperature and Moisture on the Pitch of an Organ Pipe—As the velocity of propagation is given by the relation $v = n\lambda$, any thing which changes the velocity will change the frequency or wave-length or both. In an organ pipe the length, which determines the wave-length of the note does not alter appreciably with change of temperature, but the velocity of sound increases with increase of temperature. Therefore a rise of temperature of the organ pipe increases the pitch of the note emitted by it. The presence of moisture in the air also behaves in the same way. If the air is mixed with a gas which lowers its density, the velocity increases (Arts. 43, 44) and consequently the pitch rises. If the gas increases in density the pitch of the note evidently falls.

The change in pitch of the note emitted by an organ pipe due to the presence of a foreign gas in air has found an interesting application in detecting fire damp in mines. In an apparatus two similar pipes are blown simultaneously,—one with pure air from a reservoir and other with the air of the mine. If the mine air is pure, the two tubes are in tune ; but a slight impurity affects the velocity and hence

*See Richardson's Sound p. 180

the frequency of the pipe blown with it. Beats formed between the pipes are then heard and the frequency of beats serves to measure the amount of impurity in the mine-air.

144. Tin Whistle—A tin whistle (fig. 63) demonstrates the formation of stationary waves in an organ pipe open at both ends. If all the six holes are closed with fingers and the instrument is blown with moderate pressure, the pipe emits its fundamental or lowest tone. The two ends A and A_1 (fig. i) are the seats of the antinode and there is a node situated at the middle point. If the hole furthest from the jet is opened (fig. ii) the antinode A_1 is shifted to the hole and the wave-length of the note diminishes. The note emitted rises in pitch. Similarly, the pitch rises more and more, the nearer the hole opened is to the jet. If all the holes are closed and the blowing pressure is increased, the pipe includes two intermediate nodes and one intermediate antinode (fig. iii). The note emitted is evidently the octave of that emitted in the case (i).

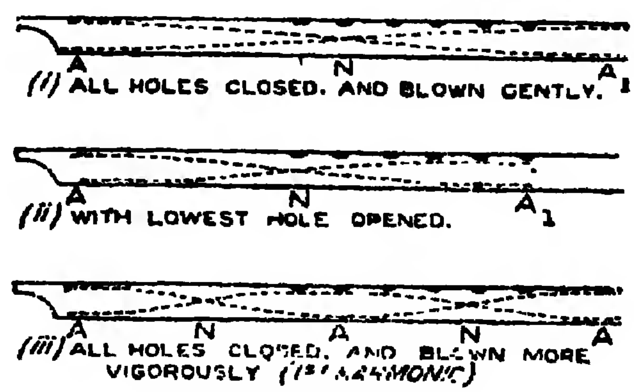

FIG. 63

145. Stringed Instruments— In a stringed musical instrument the note is emitted by the vibrations of the string. Stringed instruments are divided into three classes according to the manner in which the string is set to vibrations. These are,—*plucked string* instruments in which the note is produced by plucking the string, such as guitar, *setar* etc., (2) *struck string* instrument in which the note is produced by striking the string with a hammer, such as

pianoforte and (3) *bowed string* instruments in which the note is produced by bowing the string with a bow, such as violin, *esraj*, etc.

When a stringed instrument is excited by plucking, theoretically an infinite series of tones is produced, some of which are absent depending upon the point of plucking.* Of the series the intensity of a tone falls off rapidly with increase in order of the tone.

In a struck string instrument, such as piano, theoretically an infinite series of tones is produced. The position of the hammer is chosen such that some of the harmonics which produce dissonance are eliminated out. Further, the intensity of the harmonics falls off with increase in order of the harmonic but less rapidly than in a plucked string instrument.

Helmholtz's theory on bowed string instruments explains the formation of an infinite series of tones. The intensity of a tone falls off rapidly with increase of order of the tone. The theory† takes no account of the position of the point at which the bow is applied. But the bow-point seems to have some influence on the character of the note emitted. His experiments on the bowed string with 'vibration microscope' analyse the note produced by a violin and determine its quality.

146. Vibration Microscope—In its essential features the apparatus was proposed by Lissajous but was employed by Helmholtz to observe the vibrations of a violin string. The apparatus consists of a tuning-fork (fig. 64) driven

* See Art. 68.

† See 'Dynamical Theory of Sound', Lamb, p. 78.

electrically, one prong of which carries a small converging lens forming the objective of a compound microscope. The tube with eye-piece of the microscope is held on a rigid support. The tuning-fork is mounted horizontally and in the field of view of the microscope the violin string is placed

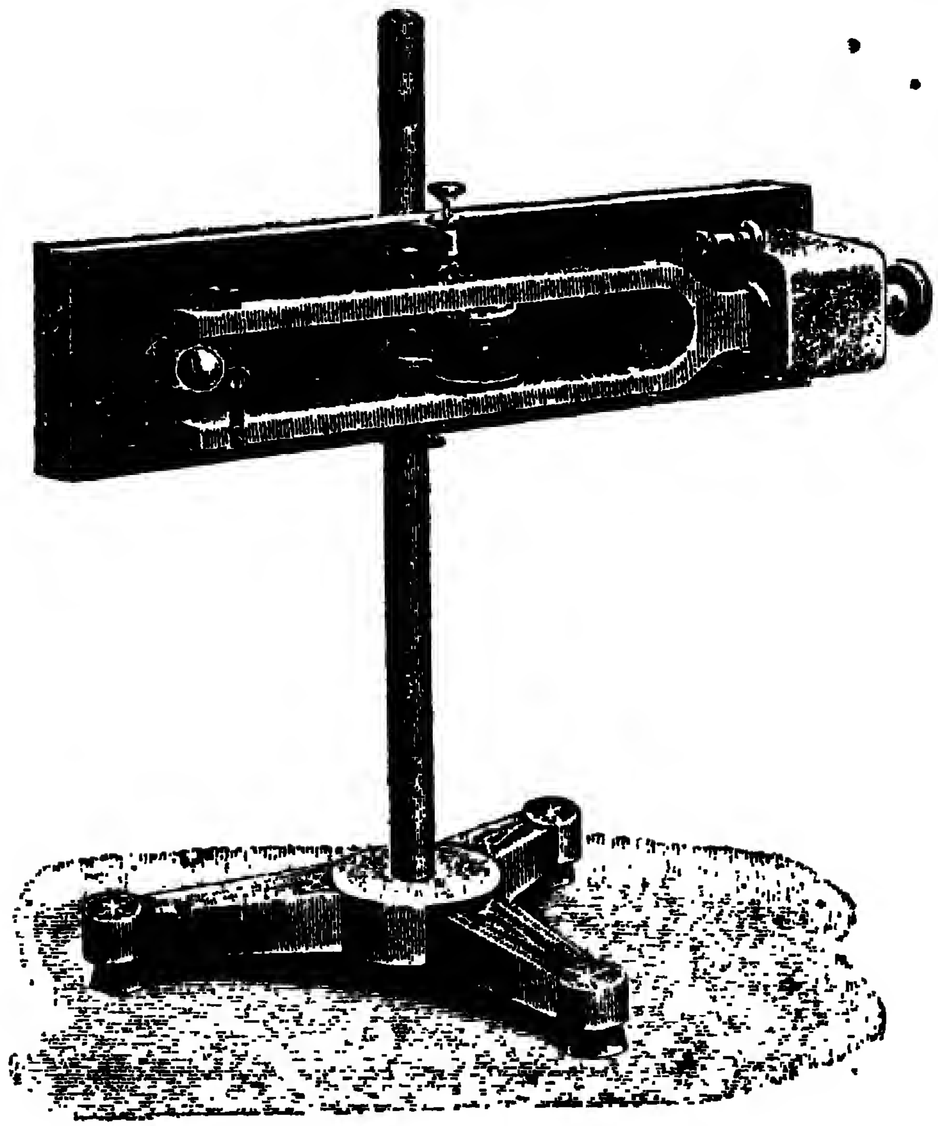

FIG. 64

in a vertical position. To view the string a starch grain was fixed on it which served as a luminous point. If the fork moves alone, the luminous point is drawn out in a bright vertical line, but if the string vibrates as well, the

observed curve is a resultant of the S. H. M.'s due to the fork and of the motion of the string. A time-displacement graph of the fork can be drawn from its amplitude and frequency of vibration. From the curve obtained by the vibration microscope the time-displacement curve of the string is obtained. Helmholtz finds that the time-displacement graph for the string consists of a two-step zigzag straight line.* By applying the Fourier analysis to the problem of the violin string, the various harmonics are determined.

147. The Phonograph—The phonograph invented by Edison is an instrument for recording and reproducing sound waves. In its modern form, it consists of a reflector, known as 'horn' (fig. 65), the narrow end of which is closed by a diaphragm. To record the sound waves a cutting tool is attached to the back of the diaphragm. The tool touches a wax cylinder. The cylinder can be rotated by clock-work and as it rotates it moves along its axis.

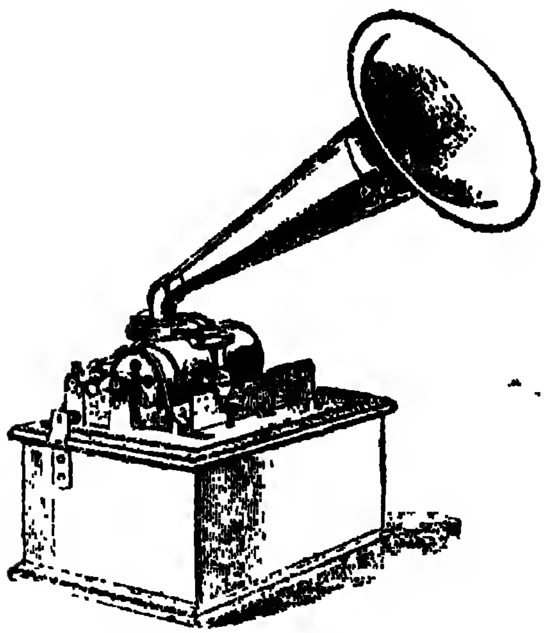

FIG. 65

When sound waves are produced before the reflector, the diaphragm with its tool is set to vibrations. The tool makes indentations of varying depth corresponding to its vibrations on the surface of the wax cylinder. To reproduce the vibrations, the cutting tool is replaced by a

* See 'Dynamical Theory of Sound', Lamb, p. 75.

rounded point and the cylinder is brought to its original position and turned as before. The rounded point with the diaphragm vibrates in a similar manner as the cylinder passes beneath it and the same sound, as spoken before it while recording, is reproduced.

In the latest type called the **gramophone**, the cylinder is replaced by a disc made of shellac, tripoli powder and other ingredients. The recording of the sound waves is first made on a wax disc and a mould of this is made of copper by electrical deposition of a thin layer of copper on the wax disc. Finally by squeezing the shellac disc on the heated copper by hydraulic press, a 'record' is made of it. The sound waves are reproduced by the 'sound box' which forms the most important part of the gramophone. The sound box consists of a lever arrangement to the shorter arm of which the 'needle' is clamped and the longer arm is attached to the centre of a mica disc. As the needle passes on the record which is rotated by a clock-work, the mica disc is set to vibration by the lever and produces alternate compressions and rarefactions in the air and reproduces the sound thereby. The shorter the needle or the portion of it outside its enclosure, the larger is the amplitude of vibration of the mica disc and louder is the sound emitted. In absence of steel needles stout hedge thorns, previously dried, have been used with success.

The above method of **acoustic recording** has many disadvantages ; one of which is that the energy available for cutting is limited to that supplied by the sound of the performer, and in consequence the performer is needed to be very close to the horn—practically an impossibility with even a moderate orchestra.

The introduction **electrical recording** has resulted in immense improvements. To receive the sound the horn and diaphragm are replaced by a high-quality microphone.* The electrical output of the microphone is amplified by means of amplifiers to a degree necessary for operating a stylus, and it is no longer necessary to crowd performers near the microphone. Further, the electric current from the microphone can be telephoned to a distance and the recording of performances in concert halls can be effected.

Examples

1. Describe the construction and operation of a gramophone. *(C. U. 1909)*

2. How can the existence of nodes and antinodes in a sounding organ pipe be demonstrated? *(C. U. 1910)*

3. Indicate the different ways in which the vibrations of the air column of an organ pipe may be started and obtained. What effect is produced in the sound emitted by an open organ pipe (*a*) by partially (*b*) by completely closing the open end of the pipe? *(C. U. 1911)*

4. Describe the distribution of nodes and loops that are formed in an organ pipe and in a string fixed at both ends. How is their existence made manifest? Tabulate the series of overtones present in these various cases. *(C. U. 1912)*

5. Describe the distribution of nodes and loops formed (*a*) in a closed organ pipe, (*b*) in a thin steel rod fixed at one end.

How are their lengths to be adjusted so that they may sound in unison?

[The velocity of sound in air $=33240$ cm. and that in steel $=5{\cdot}2\times10^5$ cm. per second.] *(C. U. 1915)*

*See Art. 154.

6. How has it been concluded that a sounding body, *e. g.* (*a*) a tuning-fork, (*b*) the air column in an organ-pipe, is in vibratory motion ?

o what physical characteristics do the loudness and the pitch of usical note correspond ? (*C. U. 1913*)

7. Describe the distribution of nodes and loops (*a*) in an open organ pipe, (*b*) in a thin rod, fixed at one end and vibrating longitudinally. How is their existence demonstrated ?

Describe a method by which the velocities of sound in various gases may be compared.

8. Describe and explain with suitable diagrams, the nature of the motion which constitutes sound.

An open organ pipe emits its fundamental note. Find its length if the velocity of sound in air $=88000$ cm. per second, and it vibrates in unison with a violin string of length 88 cm. under a stretching-force of $7{\cdot}89\times10^6$ dynes, the mass of the string per centimetre being 0·0042 gramme. (*C. U. 1916*)

9. Give a general explanation of the manner in which 'Lissajous' figures may be observed and produced and how they may be practically utilized in acoustical determinations. (*C. U. 1921*)

10. State how the presence of overtones in the sound of (*a*) a closed organ pipe and (*b*) a bell, may be observed and their pitches determined. Explain clearly the difference in the nature of the tones in the two cases. (*C. U. 1921*)

11. Clearly explain the meaning of the expression 'velocity of propagation of a longitudinal wave.' Represent graphically the state of disturbance at any time at any place during the propagation of such a wave. Detail the mechanical process involved.

Explain the formation of nodes and loops in an open organ-pipe emitting its fundamental and first overtone. (*C. U. 1924*)

12. Explain the terms —Pitch, interval, overtones, and timbre.

How would you experimentally demonstrate the existence of overtones in (*a*) a stretched string vibrating transversely, and in (*b*) a closed organ pipe ? (*C. U. 1926*)

13. Explain, with the help of neat diagrams, the methods of vibrations of an open and of a closed organ pipe.

What will happen to the pitch and wave-length of the sound emitted if the density of the air is increased by 10 per cent of its original value ? *(C. U. 1927)*

14. Describe some experimental method by means of which wave-forms of musical sound may be studied, *(C. U. 1927)*

15. Give carefully drawn diagrams showing the position of the nodes in an organ pipe, open at one end and closed at the other, for the fundamental vibration and the first three harmonics.

State the relations which exist between the lengths of closed and open organ-pipe and the pitch of the notes emitted by them.

(C. U. 1928)

16. Explain how stationary waves are produced in a closed pipe. How do they differ from progressive waves ? *(C. U. 1934)*

17. Explain how stationary waves are produced in a closed pipe. How do they differ from progressive waves ?

A closed pipe 4 ft. long and filled with a gas resounds to a given tuning-fork. If an organ pipe resounding to the same fork and containing air be 5 ft. long, what would be the velocity of sound in the gas ? The velocity in air at the temperature of the experiments is 1120 ft./sec. *(C. U. 1946)*

18. Describe the construction and operation of a gramophone. *(C. U. 1951)*

CHAPTER XVI

PHYSIOLOGICAL ACOUSTICS

148. The Human Ear—The human ear which is the organ of hearing is divided into three parts :—(1) the external ear, (2) the middle ear and (3) the internal ear.

(1) *External ear* consists of (*a*) the *pinna P.* (fig. 66), the part external to the head and (*b*) the *external auditory meatus M.* The pinna serves as a collector of sound waves.

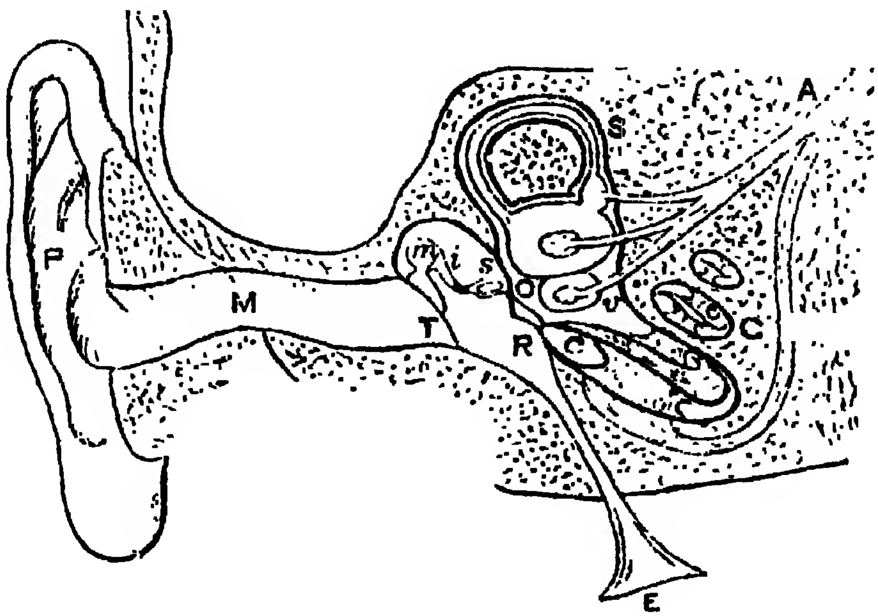

FIG. 66.

The external auditory meatus is a canal with a slight arch upwards and serves to convey the vibrating air.

(2) *Middle ear* or the *tympanum* commonly known as the drum of the ear is separated from the external auditory meatus by the *membrana tympani* or the *tympanic membrane T*, commonly known as the drum skin. The tympanum is a cavity bounded by the tympanic membrane on

the outer side and its inner side is bounded by bony walls except at two places O and R across which membranes are stretched. The membrane at O is oval and is called *fenestra ovalis* while that at R is round and is called *fenestra rotunda.* The tympanum is in communication with the upper part of the throat (pharynx) by a tube E known as the *eustachian* tube which serves to keep the air pressure on both sides of the tympanic membrane the same. A combination of three bones called ossicles extends from the tympanic membrane to the *fenestra ovalis* across the drum. The ossicles consist of (*a*) the *malleous* '*m*' or hammer bone. (*b*) the *incus* '*i*' or the anvil bone and (*c*) the *stapes* '*s*' or the stirrup bone.

(3) *Internal ear* or the *labyrinth* is the proper organ of hearing. It has a very complicated structure. The labyrinth consists of a set of cavities having bony walls. The bones forming the walls are known as the *osseous labyrinth* and the membranes within the cavities form the *membranous labyrinth.* The osseous labyrinth consists of (*a*) *vestibule* '*v*' in the outer wall of which lies the *fenestra ovalis.* It has several openings on its inner wall for the entrance of the divisions of the auditory nerve. (*b*) *Cochlea* '*c*' which is a spiral canal having the form of a snail shell. At its entrance lies the *fenestra rotunda.* The canal is divided partly by bone and partly by membrane called the *basilar membrane.* (*c*) The *semi-circular* canals S, which maintain equilibriation but do not take part in the process of hearing. The membranous labyrinth contains a fluid called *endolymph* while outside it and between it and the osseous labyrinth, is a fluid called *perilymph.*

This broadly gives the structure of the human ear.

149. Act of Hearing—The vibrations of the air caused by a source of sound are collected by the pinna and they travel through the air in the external auditory meatus. These vibrations then force the tympanic membrane into corresponding vibrations. For, the tympanic membrane is of such a size and tension that it can readily respond to any vibrations between certain wide limits. These vibrations are then communicated through the three ossicles which vibrate as if they were one to the membrane closing the *fenestra ovalis.* The vibrations of the *fenestra ovalis* are transmitted first through the perilymph on the far side of the *fenestra ovalis* and then the vibrations pass through the basilar membrane and set the endolymph of the canal of cochlea in vibrations which are ultimately transmitted to the divisions of the auditory nerves. The stimuli are conveyed by the auditory nerves to the brain and produce the perception of sound. The basilar membrane contains about 24,000 fibres. The fibres increase in length from the base to the apex of the cochlea.

According to Helmholtz's resonance theory of audition the complex sound waves of the external air are analysed by the basilar membrane. The complex sound waves travelling in the above manner set the particular fibres of the basilar membrane in sympathetic (resonant) vibration. These auditory impulses are carried to the brain by the auditory nerve fibres and give rise to a sound of corresponding loudness and pitch.

G. S. Ohm stated a law of the action of the ear. The law may be stated as follows :—The ear only experiences the sensation of a simple tone when it is excited by a simple harmonic vibration. It analyses every other periodic

vibration into a series of simple harmonic vibrations each of which corresponds to the sensation of a simple tone.

The range of hearing extends over 10 or 11 octaves ; the lowest audible tone having a frequency about 20 and the highest about 25,000. The range varies in different people and diminishes from childhood onwards.

150. The Human Voice—The organ of voice production is a box-like cartilaginous structure called the *larynx* in the front part of the throat. It is situated at the top of the wind pipe called *trachea* and below the pharynx (the upper part of the throat), the nose and the mouth.

FIG. 67.

The adjoining figure (fig. 67) shows the principal parts of the larynx concerned in the emission of voice. The larynx consists of several cartilages, which are most important for voice production. These are (1) *thyroid cartilage T*, (2) *cricoid cartilage C* and (3) *two arytenoid cartilages*.

The thyroid cartilage, also known as Adam's apple, does not form a complete ring round the larynx but only covers the front and the side portions (fig. 68). The cricoid cartilage, on the other hand, is a complete ring, the back part of which is much broader than the front (fig. 69.) On the broad back portion of the cricoid are the two arytenoid

cartilages A,A. (fig. 69). The thyroid is connected with the cricoid by a membrane which allows a revolving motion of the former within a certain range. The cricoid is connected to the arytenoids by membranes and ligaments which permit tolerably free motion between them.

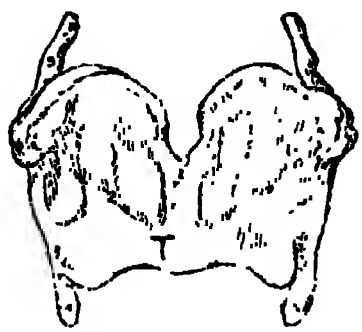

FIG. 68

FIG. 69

In the interior of the larynx lie two elastic bands called *vocal cords* (fig 70) stretching from before backwards. In the front, these are attached to the angle of the thyroid cartilage and behind to the arytenoid cartilages. These two are separated from each other by a slit-like aperture which can be increased or decreased.

FIG. 70

151. Voice Production—The human vocal instrument is compared to the reed organ pipe having two reeds. In the reed organ pipe, the note is produced by the vibrations of the reeds and a column of air. In the vocal instrument the sound is produced by the vibration of the vocal cords which correspond to two reeds, the trachea corresponding to the air chamber.

During the emission of a note, the margins of the

arytenoids are brought into contact and the edges of the vocal cords are made nearly parallel and their tension is increased. The pitch of the note emitted depends mainly upon the tension of the vocal cord. During respiration the opening between the vocal cords increases and it increases more during rapid and deep inspiration.

The vibrations of the vocal cords give the fundamentals of the note emitted, the pitch of which is determined mainly by their tension. The throat, mouth and nose supply resonating cavities, the shape and size of which the speaker can vary at will. These resonating cavities supply suitable overtones which determine the quality of the sound emitted.

The vocal cords are much longer in men than in women and the wave-length of the sound emitted by a man is much longer than that emitted by a woman. This makes the pitch of the sound emitted by a woman higher than that of a man.

152. Vowel Sounds.—The consonants of a speech are produced by interruptions, more or less complete, of the outflowing air in different situations of the tongue, teeth, lips, etc. The vowel sound is produced by suitably altering and adjusting the shape and size of the resonating cavities of throat, mouth and nose so as to give rise to certain overtones peculiar to the quality of a particular vowel, the fundamental being produced by the vocal cords.

There are two theories prevalent on the vowel sound. One is the '*fixed pitch theory*' of Willis (1832). Willis carried out experiments in 1829 to produce vowel sounds artificially by a reed and resonant cavities. From these experiments he was led to conclude that whatever the pitch of the fundamental on which the vowel is spoken may be, the resonant cavities introduce overtones having definite

frequencies for a particular vowel. This theory was modified by Hermann from his researches with phonograph. According to Hermann's view the pitch of the overtones might vary a little without altering the character of the vowel and his theory is called the '*inharmonic theory*.'

The other theory of the vowel production is the '*relative pitch theory* or *harmonic theory* of Helmholtz, according to which the vowel quality is determined by the overtones having fixed ratios of frequency to that of the fundamental. Experiments with phonograph to decide between the two theories is not very conclusive, but tend to support Hermann's view. Stewart has recently (1922) reproduced vowel sounds by transient electrical oscillations and remarks that the distinction between Willis' theory and Helmholtz's theory is not serious. There is general agreement between the two sets of data.

A comprehensive investigation of speech sound has more recently been made by Crandall in 1925 with microphone and valve amplifier. He obtained oscillograph records of the various vowels and consonants. Analysis of the records yields much fundamental information relating to speech sounds.*

Examples

1. Compare the actions of the phonograph and telephone†, and explain the characteristics of the vocal organ in man. *(C. U. 1924)*

2. What is it that enables us to distinguish the sounds of the different vowels when we hear them, and how do their differences arise ? *(C. U. 1931)*

3. Write a short note on Human ear. *(C. U. 1915, 1947)*

4. Describe the mechanism of the human ear with regard to the production of sensation of sound. Explain what you know about Helmholtz's resonance theory of hearing. *(C. U. 1949)*

* See 'A Text-book of Sound', Wood.

† For telephone, See Art. 158.

CHAPTER XVII

ELECTRIC TRANSMISSION OF SOUND

153. Telephone—In 1876, Graham Bell invented the speaking telephone. The invention was favourably reported on by Lord Kelvin at the British Association in 1876. The action of Bell's telephone both as a receiver and transmitter of sound will be understood from fig. 71.

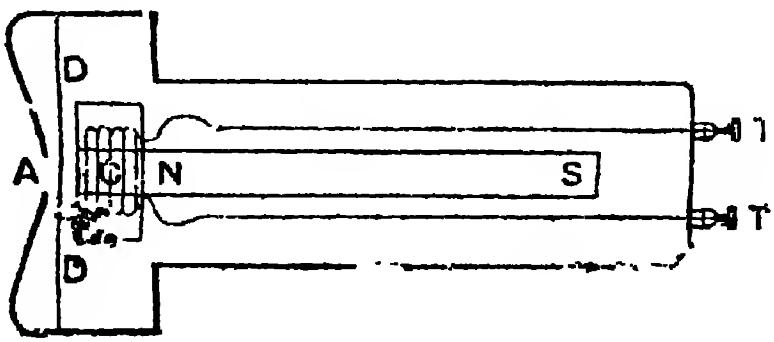

FIG. 71

In this figure NS is a permanent bar magnet of steel. One pole of the magnet is turned towards a vibrating plate D made of a very thin sheet of iron and is known as the diaphragm. Before the diaphragm there is a saucer-shaped mouthpiece A. Round the end of the magnet next to the diaphragm, a coil of insulating wire C is wound, whose ends are connected to the outside terminals TT. To telephone a message two such instruments, one at the sending station and the other at the receiving station, are connected by a pair of wires, or one terminal of each instrument may be 'earthed', and the remaining terminals connected by a single wire.

When someone speaks before the mouthpiece, the sound waves falling upon it were concentrated on the diaphragm

and set it in vibration. A change in the position of the diaphragm changes the magnetic field between it and the magnet and thereby a transient induced current is sent through the coil and the wires connected to the coil, called the 'line.' An approach of the diaphragm strengthens the field and sends a current one way, while a recession of the diaphragm weakens the field and sends the current in the reverse way. Further these currents are proportional in magnitude to the motion of the diaphragm. Thus the features of the spoken sounds are represented by *undulatory currents* of varying magnitude started by the transmitter.

These currents are propagated along the line and in passing through the coil in the receiving instrument serve, according to their direction to increase or decrease the magnetisation of the magnet of the receiving instrument. The diaphragm before the magnet is thus more or less strongly attacted by the magnet than when no current is passed. The diaphragm is thus set to similar vibrations as those of the diaphragm of the sending station producing similar sound waves in air as received by the instrument of the sending station.

In the arrangement just described no battery is needed as the transmitter acts as a generator and plays the function of a tiny dynamo. Lord Rayleigh determined that the Bell receiver would respond to a minimum current of the order of $4{\cdot}4 \times 10^{-8}$ ampere. Tait found that the current to which the receiver can respond is 2×10^{-12} ampere.

154. Microphone—In 1878 Professor Hughes invented a device for the transmission of vibrations. The instrument, though apparently rough, is of surprising delicacy and is known as *microphone*. In fig. 72, a modified form of Hughes' original microphone is shown. A small pencil *A* of gas

carbon with pointed ends rests lightly and vertically in small circular holes drilled in two pieces of gas carbon B and C. The pieces of gas carbon B and C are fixed to a thin sound-

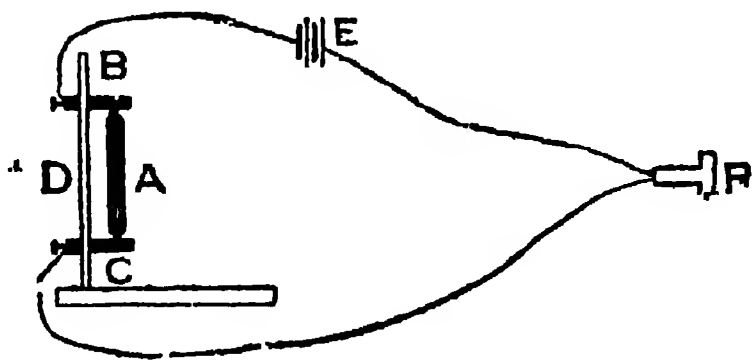

FIG. 72

ing board D mounted on a firm base. A battery E and a telephone receiver R, described in the last article, are included in the electric circuit with B and C.

When someone speaks before the sounding board, the vibrations of A introduce alterations in resistance between the carbon pieces B, C and the pencil A. This produces changes in the current through the coil of the telephone receiver R and as a consequence the magnetisation of the magnet in the telephone receiver fluctuates. The diaphragm is set to vibrations and emits a sound corresponding to the spoken sound.

155. Loud-Speaker—Many loud-speakers of different types such as 'moving coil,' 'moving iron' and 'moving conductor' types are invented. A moving coil type is preferable primarily for public address in large halls and for use in conjunction with a large horn.

Wente and Thuras described a moving coil loud-speaker in 1931. Fig. 73 shows in section a 'dynamic' or 'moving coil' loud-speaker. N, S. represent the pole pieces of an electromagnet excited by a source of direct current. A

coil of wire *cc*, known as the 'speech coil', is free to move in the narrow gap between the poles of the electro-magnet. The speech coil is attached to a large light non-magnetic diaphragm *D* which is made of stiff paper in

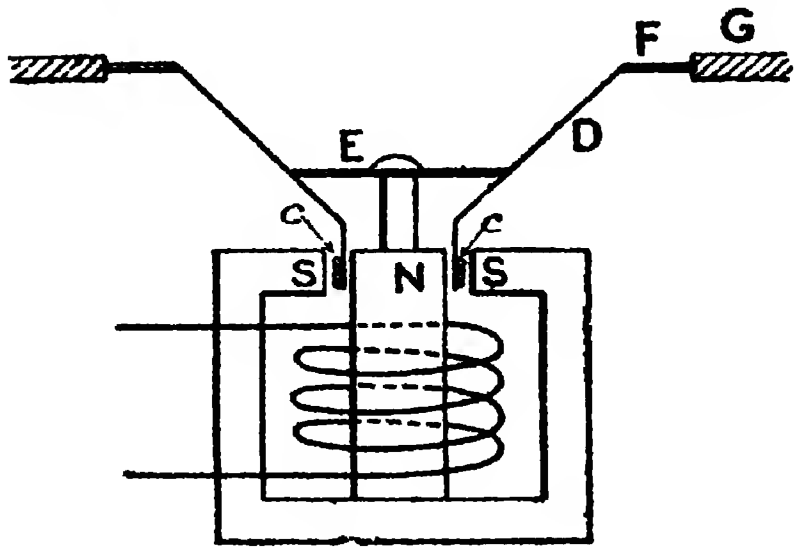

FIG. 73

the form of a hollow cone. Sometimes the diaphragm is a light disc of Balsa wood which is relatively too powerful for higher frequencies. Aluminium has been used as the material of the conical diaphragm in order to provide greater mechanical stiffness. The movement of the speech coil is made possible by floating the diaphragm to which the coil is attached on flexible ring-type support *E*. The diaphragm at its periphery is attached to a flexible support *F* which in its turn is connected to a frame *G*.

There are two distinct varieties of the dynamic loud-speaker :—(1) those which do, and (2) those which do not employ some type of horn.

The current produced by the speech in the microphone is amplified by suitable amplifiers and is led to the speech coil. The varying speech currents through the coil set up a correspondingly changing field which reacts with the

magnetic flux between the magnet pole pieces. The whole diaphragm is thus caused to vibrate in the corresponding manner and produces the sound.

156. Talking Picture—As in the phonograph, the talking film involves two distinct processes of recording and reproduction. There are two methods of recording and reproduction the principles of which are briefly described below.* For a detailed study of talking film, a book on the subject may be read.

(1) *Optical Methods of Recording and Reproduction*—Sound waves are often recorded photographically for this purpose by means of changes of intensity of a beam of light which affects the density of blackening of a continuously moving photographic film, in a manner which corresponds exactly to the pressure variations of the sound wave being recorded. There is also another method in which they are recorded upon a film as a trace of constant intensity but variable width. The former is known as the *variable density system* and the latter, the *variable area system.*

In the variable density system of recording, a lamp of constant brilliance is employed, but the light passes to the film through a slit the width of which varies according to the speech currents. One form of variable slit or 'light valve' is essentially an electro-magnetic shutter. Currents from the microphone and amplifier produced by spoken sound flow in the loop of the shutter and cause it to open or close according to the current vabrations. The film thus receives a varying exposure.

In the variable area system of recording, a narrow beam of light of constant intensity is reflected from the mirror of an oscillograph of the Duddell type. The beam of light

* Summarised from Davis' Modern Acoustics.

then passes through a slit of fixed dimensions and is focussed on the film. Sound currents flow through the oscillograph and cause the mirror to vibrate according to the current variations. The resulting sound track on the film is of uniform density but has the appearance of a serrated edge covering correspondingly varying area. The film thus obtained in either way is called a **sound film**.

For reproducing speech or music from the sound film, the film runs along one side of the picture. Light from a bright filament lamp is focussed on the sound film on the other side of which a light sensitive cell is situated. Selenium cells or better photo-electric cells are used for this purpose with advantage. The photo-electric cell (fig 74) consists of an evacuated glass bulb in which a plate and a wire are enclosed. The plate is made of some durable metal with a thin film of sodium, potassium or cæsium deposited on it to make it sensitive to ordinary light. The plate and the wire are connected respectively to the negative and positive poles of a battery. When light falls on the plate, electrons are emitted from the plate depending on the light falling and these electrons are attracted by the positively charged wire. The loss of electrons from the plate is, however, at once made up by the battery. The photo-electric current through the cell thus depends on the light falling on the plate. At any instant the sensitive cell is illuminated to a degree depending upon the photogra-

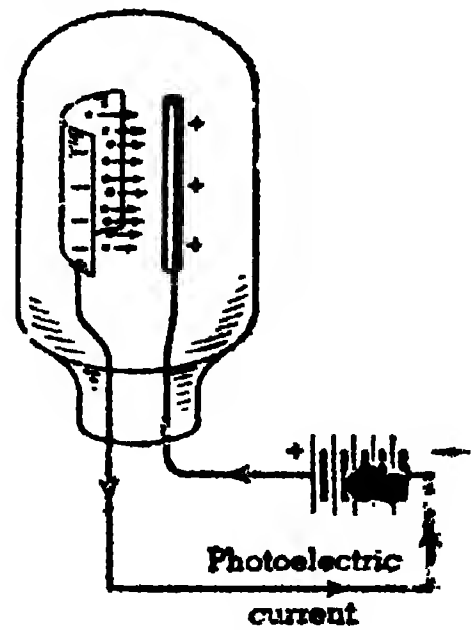

FIG. 74

phic density or area (as the case may be) of the sound film record at the illuminated region. The varying electrical current thus produced is suitably amplified ; it then actuates a loud-speaker system and the corresponding speech or music is produced.

The arrangement for reproduction is shown in fig. 75.

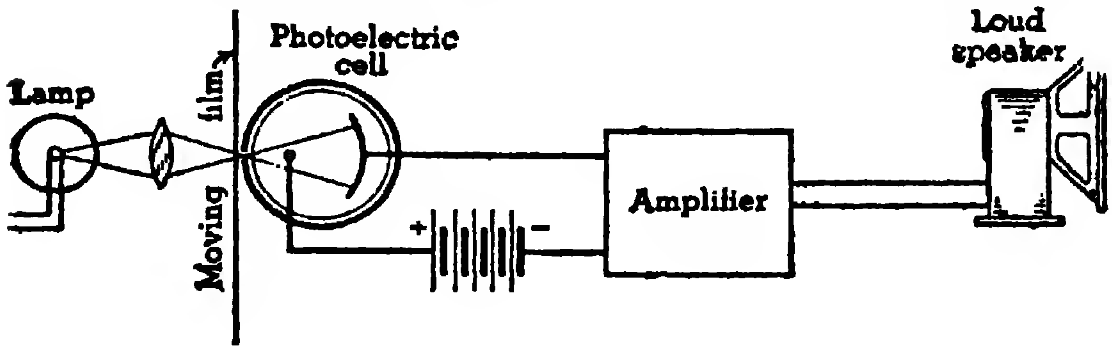

FIG. 75

(2) *Electro-magnetic Methods of recording and Reproduction*—In the electro-magnetic method, which is more convenient than the optical method, sound film is produced on a thin steel wire or ribbon. The wire is passed between the poles of an electro-magnet. The speech currents pass through the coils of the electro-magnet, the magnetic field of which thereby varies according to the fluctuations of the speech currents. The varying magnetic field produces transverse magnetisation of the wire of varying intensity along its length. At any point on the wire the degree of residual magnetisation depends on the strength of the magnetic field at the moment the point passes between the poles of the magnet Thus after the recording operation is over, the transverse magnetisation of the wire fluctuates along its length according to the variations of the speech currents.

To reproduce the sound, the wire is passed between the poles of an electro-magnetic reproducer. The flux through the reproducer varies according to the fluctuations in magnetisation of the moving wire-record and this sets up corresponding currents in the coils of the reproducer. These currents are amplified and passed through a loud-speaker which produces the corresponding speech or music.

Great progress has been made by using a steel tape record of high magnetic coercivity, and the quality of reproduction is almost as good as that from a gramophone record.

CHAPTER XVIII

LONGITUDINAL VIBRATIONS OF RODS AND BARS : SUPERSONICS

157. The velocity of propagation of longitudinal waves in a solid is given by

$$V=\sqrt{\frac{Y}{\rho}} \quad \ldots \quad \text{(See Arts. 58 and 59)}$$

The frequency of vibration is given by

$$n\lambda = V$$

$$\therefore \quad n\lambda = \sqrt{\frac{Y}{\rho}}$$

$$\text{or} \quad n = \frac{1}{\lambda}\sqrt{\frac{Y}{\rho}}.$$

Case I. *Rod clamped at one end*—The clamped end must be a node, and the free end an antinode of the stationary vibrations set up in the rod. In the fundamental vibration of the rod its length $l=\frac{\lambda}{4}$ and the frequency of fundamental vibration is given by

$$n_1 = \frac{1}{4l}\sqrt{\frac{Y}{\rho}}$$

For higher modes of vibration $l=\frac{3}{4}\lambda$, $\frac{5}{4}\lambda$ etc.

and the frequencies are $n_2 = \frac{3}{4l}\sqrt{\frac{Y}{\rho}}$, $n_3 = \frac{5}{4l}\sqrt{\frac{Y}{\rho}}$, etc.

Case II. *Rod free at both ends*—Each end is the seat of an antinode. The length l of the rod for fundamental vibration $=\frac{\lambda}{2}$. The frequency of fundamental vibration is given by $n_1 = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}$.

For higher modes of vibration $l=\lambda$, $\frac{3}{2}\lambda$, etc.

and the frequencies are $n_2 = \frac{1}{l}\sqrt{\frac{Y}{\rho}}$, $n_3 = \frac{3}{2l}\sqrt{\frac{Y}{\rho}}$, etc.

Case III. *Rod clamped at the middle*—The middle point is a node and the ends are antinodes. The length l of the rod for fundamental vibration is given by

$$n_1 = \frac{1}{2l}\sqrt{\frac{Y}{\rho}}.$$

For higher modes of vibration $l = \frac{3}{2}\lambda, \frac{5}{2}\lambda$, etc,

and the frequencies are $n_2 = \frac{3}{2l}\sqrt{\frac{Y}{\rho}}$, $n_3 = \frac{4}{2l}\sqrt{\frac{Y}{\rho}}$, etc.

The frequencies of longitudinal vibrations of rods observed in the foregoing relations are subject to the conditions that the diameter of the rod is always small compared to half-wave-length of the. waves and that the change of lateral dimension is negligible.

The frequency of longitudinal vibrations in bars is generally very large compared to that of transverse vibrations. In case of a string it will be found that no available tension can make transverse waves travel as fast as the longitudinal waves through it.*

158. Excitation of Longitudinal Vibrations—The excitation of longitudinal vibrations may be made in a number of ways stated in connection with Kundt's tube experiment (Arts. 97 and 98). The chief among these are—

(1) *Rubbing*—Bars made of metal or wood clamped at the middle may be set to longitudinal vibrations by steady frictional drag of a resined cloth drawn with a moderate pressure along the bar towards the antinode. Fine sand sprinkled on the bar will show the positions of nodes and antinodes as in Kundt's experiment.

(2) *Striking*—Short and stiff bars may be set to longitudinal vibrations by striking one end of them with a

* See Art. 66.

* See Art. 97.

hammer. This method often shows complex results due to transverse and longitudinal vibrations.

(3) *Electro-magnetic*—Bars of magnetic material may be set to powerful longitudinal vibrations with help of currents of resonating frequency. With bars of non-magnetic material a thin disc of soft iron may be cemented firmly to one end of it. The fundamental and harmonics can be obtained readily pure and free from transverse vibrations.

(4) *Electrostatic*—To apply the electrostatic method of excitation, the bar is clamped at the middle. It is placed close to the plane surface of a massive block of metal the opposing faces forming a condenser which is connected directly with the tuned circuit of an oscillating valve.

159. Piezo Electricity—The electrical effects produced by pressure on asymmetric crystals was discovered by J. and P. Curie in 1880. They found that when a mechanical pressure was applied on certain crystals, there was a difference of electric potential in a perpendicular direction which gave rise to an electric current in a circuit connected with the faces of the crystal. If, however, a tension is applied on the crystal, the direction of the electric current is reversed.

That this effect is reversible was predicted by Lippmann in 1881. If a voltage is applied to the faces of the crystal, it produces corresponding changes in dimensions. The effects are best observed in slices cut in certain directions from the natural crystals. The phenomenon is shown by a large number of crystals of which the best known examples are quartz, tourmaline and Rochelle salt.

160. Resonant Vibrations in Crystal Slices—If an alternating difference of potential be applied to two opposite faces of a quartz crystal, there will be rapid alternations of

compressions and extensions in two other perpendicular directions. If one of these directions is extended, the other direction will be compressed and *vice versa*. The two strains are such that there is no change in the volume of the crystal. It will be observed that these compressions and extensions in either direction constitute longitudinal vibrations whether they are along the length or thickness of the crystal.

The longitudinal forced vibrations of the crystal will be usually insignificant. But when the frequency of the forced vibration in either direction coincides with the natural frequency of vibration in that direction, the vibrations of the crystal will be prominent due to resonance phenomena.

With quartz crystals, the velocity of propagation of longitudinal waves, which depends on the particular axis chosen, has however nearly the same value in different directions. The density of quartz is 2·654 gms. per c.c, and the average value of Young's modulus is 8×10^{11} dynes per sq. cm. Hence the average value for the velocity of longitudinal wave propagation is given by

$$V=\sqrt{\frac{Y}{\rho}}=\sqrt{\frac{8\times10^{11}}{2\cdot654}}=5\cdot5\times10^{5}\ \text{cms. per sec.}$$

and the frequency $n=\dfrac{5\cdot5\times10^{5}}{\lambda}$

If both faces of the crystal slice are free to move, the fundamental mode of vibration will have a nodal plane in the middle of the slice and the faces will correspond to antinodes of the stationary vibrations in the slice. The thickness t of the slice will be $\lambda/2$. The frequency of fundamental vibration will be given by

$$n=\frac{5\cdot5\times10^{5}}{2t}\ \text{cycles per sec.}$$

161. Supersonics—Supersonic or ultra-sonic waves are those produced by the longitudinal vibration of crystals. They have frequencies far above the audible limit. The circuit* used by Pierce to produce such waves is shown in

* Summarised from Alexander Wood's 'Acoustics'.

fig. 76. One electrode of the crystal vibrator C is connected with the plate P of the valve and the other electrode to the grid G. The filament F is heated by a battery and the plate

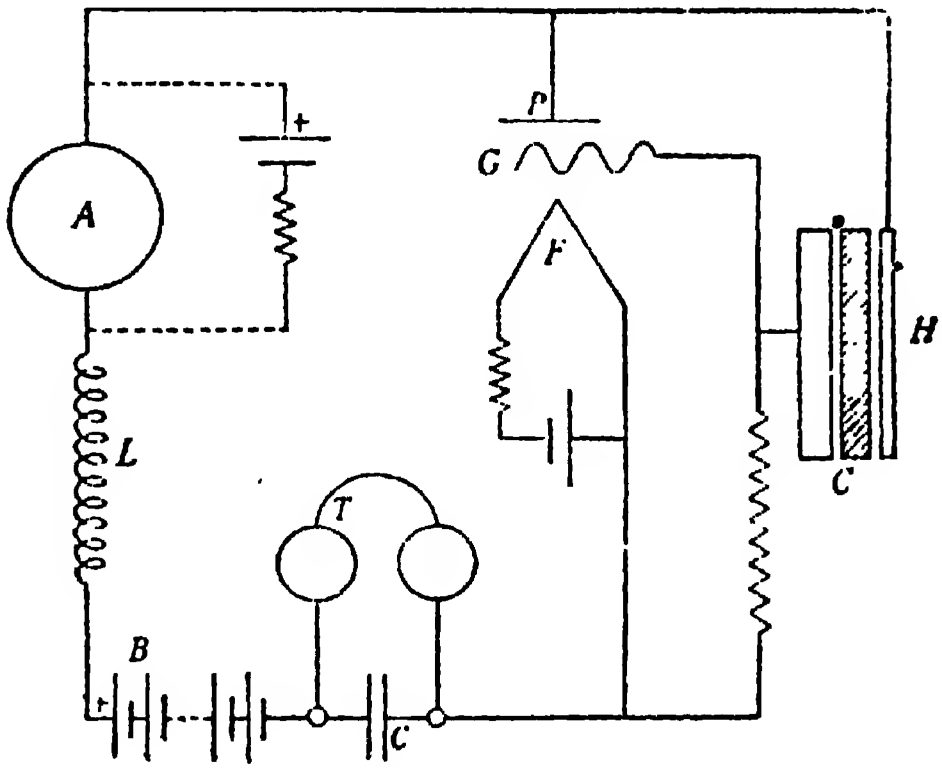

FIG. 76

current is supplied by another battery B. The plate circuit also includes a micro-ammeter A and a telephone T shunted by a condenser C. The plate circuit also includes a resistance L of about 30,000 ohms or a large inductance of about 20 millihenries. This system produces oscillations in the circuit and the mechanical vibrations of the crystal with a frequency equal to the natural frequency of vibration parallel to its thickness. An aperture H in the front electrode allows the train of waves to emerge.

Supersonic waves have been used for a great variety of purposes. They are applied in transforming immiscible liquids, such as water and oil into homogenious stable emulsion. Supersonic waves have been applied by Langevin to the development of supersonic depth founder. They are used not only for ordinary depth determinations but for detecting sunken obstacles.

INDEX

[*References are to Articles. Names of persons are in italics.*]

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